Student Name:
Student Id#:

McGILL UNIVERSITY

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATH 381

COMPLEX VARIABLES AND TRANSFORMS

Examiner: Professor J. Toth
Associate Examiner: Dr. Axel Hundemer

Date: Tuesday December 8, 2009.
Time 2:00 PM- 5:00 PM

INSTRUCTIONS

1. Please answer all questions in the exam booklets provided.
2. This is a closed book exam.
3. Use of a regular and/or translation dictionary is not permitted.
4. Calculators are not permitted.
5. This examination paper been must be handed in with your exam booklet.

This exam comprises of the cover page and 1 page of 8 questions

MATHEMATICS 381 FINAL EXAMINATION

Each question is worth 10 points. Please show all your work.
All contours are positively oriented.

1) Compute the contour integral

$$\int_{|z|=1} \frac{e^{i(1+z)}}{z^{10}}\, dz.$$

2) Use residues to compute

$$\int_0^{2\pi} \frac{6}{4+\sin\theta}\, d\theta.$$

Clearly indicate the contour you are using and justify all your steps.

3) Determine the Cauchy principal value of the integral

$$\int_{-\infty}^{\infty} \frac{\sin x}{(x^2+2)^2} dx.$$

Clearly indicate the contour you are using and justify all your steps.

4) (a) Determine the values of $(x, y) \in \mathbb{R}^2$ for which $f(z) = e^x(y + ie^y)$ has a complex derivative $\frac{df}{dz}$ where $z = x + iy$.

5) Expand $f(z) = \frac{z}{z-i}$ in a Laurent series centered at $z_0 = 1$ and determine the annulus of convergence.

6) Compute

$$\mathrm{Res}_{z=k} \frac{1}{\sin(\pi z)},$$

where k is an integer.

7) Compute the inverse Laplace transform $f(t) = \mathcal{L}^{-1}F(s)$ where $F(s) = \frac{s}{s^2-4}$. Clearly indicate the contour used.

8) Let $u(x, y)$ be a harmonic function that is *radial*. Recall that a function is radial if it is constant on all concentric circles centered at $(0, 0)$. Write down the general formula for $u(x, y)$. Please justify your answer. (Hint: use polar variables).