

Student Name:
Student Id#:

McGILL UNIVERSITY

FACULTY OF ENGINEERING

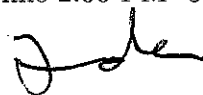
FINAL EXAMINATION

MATH 381

COMPLEX VARIABLES AND TRANSFORMS

Examiner: Professor J. Toth
Associate Examiner: Dr. Axel Hundemer

Date: Tuesday December 8, 2009.
Time 2:00 PM- 5:00 PM



INSTRUCTIONS

1. Please answer all questions in the exam booklets provided.
2. This is a closed book exam.
3. Use of a regular and/or translation dictionary is not permitted.
4. Calculators are not permitted.
5. This examination paper must be handed in with your exam booklet.

This exam comprises of the cover page and 1 page of 8 questions

MATHEMATICS 381 FINAL EXAMINATION

Each question is worth 10 points. Please show all your work.
All contours are positively oriented.

- 1) Compute the contour integral

$$\int_{|z|=1} \frac{e^{i(1+z)}}{z^{10}} dz.$$

- 2) Use residues to compute

$$\int_0^{2\pi} \frac{6}{4 + \sin \theta} d\theta.$$

Clearly indicate the contour you are using and justify all your steps.

- 3) Determine the Cauchy principal value of the integral

$$\int_{-\infty}^{\infty} \frac{\sin x}{(x^2 + 2)^2} dx.$$

Clearly indicate the contour you are using and justify all your steps.

- 4) (a) Determine the values of $(x, y) \in \mathbb{R}^2$ for which $f(z) = e^x(y + ie^y)$ has a complex derivative $\frac{df}{dz}$ where $z = x + iy$.

- 5) Expand $f(z) = \frac{z}{z-i}$ in a Laurent series centered at $z_0 = 1$ and determine the annulus of convergence.

- 6) Compute

$$\operatorname{Res}_{z=k} \frac{1}{\sin(\pi z)},$$

where k is an integer.

- 7) Compute the inverse Laplace transform $f(t) = \mathcal{L}^{-1}F(s)$ where $F(s) = \frac{s}{s^2-4}$. Clearly indicate the contour used.

- 8) Let $u(x, y)$ be a harmonic function that is *radial*. Recall that a function is radial if it is constant on all concentric circles centered at $(0, 0)$. Write down the general formula for $u(x, y)$. Please justify your answer. (Hint: use polar variables).