NAME (underline family name):

STUDENT NUMBER: SIGNATURE:

FINAL EXAMINATION

323

PROBABILIY

Examiner: D. A. KELOME **Date:** December, 7 2006
Associate Examiner: M. ASGHARIAN **Time:**2:00 pm - 5:00 pm

Instructions

1. Write your name and student number on this examination script.
2. **No books, calculators or notes allowed.**
3. This examination booklet consists of this cover, 7 pages of questions and 2 blank pages (10 numbered pages in all). Please take a couple of minutes in the beginning of the examination to scan the problems. (Please inform the invigilator if the booklet is defective.)
4. Answer all questions. You are expected to show all your work. All solutions are to be written on the page where the question is printed. You may continue your solutions on the facing page. If that space is exhausted you may continue on the blank pages at the end, clearly indicating any continuation on the page where the question is printed.
5. Your answers may contain expressions that cannot be computed without a calculator.
6. Circle your answers where confusion could arise.
7. You may use regular and translation dictionaries

GOOD LUCK!

Score Table

Problem	Points	Out of
1.		10
2.		10
3.		10
4.		10
5.		10
6.		10
7.(Bonus).		
Total:		60

Question 1. Suppose the distribution of grades in a certain class is given by the following density (pdf):

$$f(x) = \begin{cases} \frac{x}{2500} & \text{if} \quad 0 \leq x \leq 50 \\ \frac{1}{25} - \frac{x}{2500} & \text{if} \quad 50 \leq x \leq 100 \\ 0 & \text{otherwise} \end{cases}$$

(a) What is the probability that one given student will pass the course (Assume the passing grade is 70)(*3 marks*)

(b)What is the expected grade of any given student.(*4 marks*)

(c) If there are 70 students in the class what is the expected number of students who will pass the course.(*3 marks*)

Question 2. Two components of a computer have the following joint pdf for their useful lifetimes X and Y.

$$f(x,y) = \begin{cases} xe^{-x(1+y)} & \text{if } 0 \leq x \text{ and } 0 \leq y \\ 0 & \text{otherwise} \end{cases}$$

(a) What is the probability that the lifetime X of the first component will exceed 3?(*3 marks*)

(b) What are the marginal pdf's of X and Y? Are the two lifetimes independent?(*4 marks*)

(c) What is the probability that the lifetime of at least one component exceeds 3?(*3 marks*)

Question 3. Suppose that your waiting time for a bus in the morning is uniformly distributed on $[0, 8]$, whereas waiting time in the evening is uniformly distributed on $[0, 10]$ independent of morning waiting time.
(a) If you take the bus each morning and evening for a period of five days, what is your total expected waiting time? *(2 marks)*

(b) What is the variance of your total waiting time. *(2 marks)*

(c) What are the expected value and variance of the difference between morning and evening waiting time for a particular day. *(3 marks)*

(d) What are the expected value and the variance of the difference between the total morning waiting time and total evening waiting time for that 5 day period? *(3 marks)*

Question 4. A merchant can stock a perishable item. The daily demand of this perishable item is represented by a random variable X($X$is assumed to be discrete) . Suppose that the probability function of X is

$$P(X=k)=\begin{cases} Ck & \text{if } \ k=1,2,3 \\ 0 & \text{otherwise} \end{cases}$$

where C is a fixed positive constant.
(a) Find C (*3 marks*)

(b)The merchant buys each item for 2$ and can sell it for 6$. if any item is left at the end of the day, it is considered a total loss. Find the optimal number of items the merchant should stock in order to maximize his expected daily profit.(*7 marks*)

Question 5.

(a) Derive the moment generating function of a Gamma distribution with parameters α, β. State clearly the interval of definition.(*4 marks*)

(b) Let $N(t)$ represents the number of customers arriving at a bank in the interval $[0, t]$. Assuming that $N(t)$ is a Poisson process with rate λ, find the distribution of the arrival time U_n of the nth customer.(*6 marks*)

Question 6. Suppose X has an exponential distribution with mean equal to β. Conditioned on $X = x$, Y has a uniform distribution on the interval $[0, x]$.

(a) Derive an expression for the marginal distribution of Y.(Hint: an integral expression is fine.)(*4 points*)

(b) Find the mean and variance of Y.(*6 points)*)

Bonus Question. *Let* X_1*,* X_2*,.....,* X_n *be random variables denoting* n *independent bids for an item that is for sale. Suppose each* X_i *is uniformly distributed on the interval* $[100, 200]$*. If the seller sells to the highest bidder, how much can he expect from the sale?*

Continuation page(Please refer to the question)

Continuation page(Please refer to the question)