Last Name:..................................

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McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATH 315

Ordinary Differential Equations

Examiner:Dr.M. Alakhrass
Associate Examiner: Professor GowriSankaran

Date: Dec 10, 2009.
Time: 9-12 am

INSTRUCTIONS

1. Please answer all the eight questions.
2. This is a closed book exam.
3. Calculators are not permitted.
4. A table of Laplace transforms is supplied.
5. Translation dictionaries are permitted.

This exam booklet consists of this cover page, 8 pages of questions and 2 blank pages

1. Solve the differential equation:

$$y' = \frac{3y^2 + 2xy}{2xy + x^2}.$$

2. By finding integrating factor of the form $x^p y^q$ solve the initial value problem

$$(3 - 20x^2y)dx + (\frac{2x}{y} - 12x^3)dy = 0 \text{ with the initial condition } y(1) = 2.$$

3. Solve the differential equation:

$$2y^2y'' + 2y(y')^2 = 1.$$

4. Find the general solution of

$$x^2y'' - 2xy' + 2y = x^3e^x.$$

5. Find the general solution $y(x)$ of

$$y^{(4)} - 6y^{(3)} + 9y'' = x.$$

6. **(a)** Find the Laplace transform of the following functions:

   (i) $f(t) = \begin{cases} \sin t, & 0 \le t < \pi; \\ \sin(t-\pi), & t \ge \pi \end{cases}$

   (ii) $g(t) = \int_0^t \tau \cos(2\tau) e^{4(t-\tau)}\, d\tau$

   (a) Find the inverse Laplace transform of the function: $F(s) = \frac{(s-2)e^{-s}}{s^2-4s+3}$

   (b) Use Laplace Transform to find the solution of the initial value problem:

$$y'' - 2y' + 5y = \delta(t-1), y(0) = 0, y'(0) = 0$$

Table of Laplace Transforms	
$f(t)$ for $t \ge 0$	$F(s) = \mathcal{L}(f)$
t^n	$\frac{n!}{s^{n+1}}$ $(n = 0, 1, \ldots)$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$u_a(t)$	$\frac{e^{-as}}{s}$
$\delta(t-a)$	e^{-as}
$f'(t)$	$s\mathcal{L}(f) - f(0)$
$f''(t)$	$s^2\mathcal{L}(f) - sf(0) - f'(0)$
$e^{at}f(t)$	$F(s-a)$
$u_a(t)f(t-a)$	$e^{-as}F(s)$

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7. Solve the following equation by means of a power series about the point $x = 0$.

$$y'' - y = 0.$$

8. Using Frobenius method, a non trivial solution of the differential equation

$$x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0$$

has the form $y_1 = x^r \Sigma_{n=0}^{\infty} a_n x^n$.

(a) Show that $x = 0$ is a regular singular point.

(b) Find the indicial equation and its roots.

(c) Find the recurrence relation for the coefficients

(c) Solve the recurrence relation for the coefficients using the large root (r_1) of the indicial equation.

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