McGILL UNIVERSITY

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATH 264

ADVANCED CALCULUS

Examiner: Professor J.J Xu Date: Monday December 17, 2007
Associate Examiner: Jeremy Van-Horn Morris Time: 2:00 PM- 5:00 PM

INSTRUCTIONS

1. Please answer questions in the exam booklets provide.
2. This is a closed book examination. No books, crib sheets or lecture notes permitted.
3. Calculators are not permitted.
4. Use of a translation dictionary is permitted. No other types of dictionaries are permitted.

This exam comprises the cover page, two page of eight questions.

Final Examination of Math-264 Advanced Calculus

(December 2007)

(1) Evaluate the double integral

$$\iint_D xy\,dA$$

where D is the triangular region with vertices $(0,0)$, $(2,0)$ and $(0,6)$.

(2) Find the volume of the region between the two paraboloids:

$$\begin{aligned} (S_1): &\quad z = 10 - x^2 - y^2; \\ (S_2): &\quad z = 2(x^2 + y^2 - 1). \end{aligned}$$

(3) Find the work done by the force field

$$\mathbf{F} = (y^2 \cos x + z^3)\mathbf{i} + (2y \sin x - 4)\mathbf{j} + (3xz^2 + 2)\mathbf{k}$$

in moving a particle along the curve

$$\{C_1\} : \begin{cases} x = \sin^{-1} t \\ y = 1 - 2t \\ z = 3t - 1 \end{cases} \qquad (0 \le t \le 1),$$

from point $(0, 1, -1)$ to the point $(\frac{\pi}{2}, -1, 2)$ and then returning along the straight line C_2 from $(\frac{\pi}{2}, -1, 2)$ to $(0, 1, -1)$.

(4) Compute the flux of the vector field $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ upward through the part of the surface $z = 5 - x^2 - y^2$ lying above the plane $z = 1$.

(5) Let (D) be the region $x^2 + y^2 + z^2 \le 4a^2$, $x^2 + y^2 \ge a^2$. The surface (S) consists of s cylindrical part (S_1) and a spherical part (S_2). Evaluate the flux of

$$\mathbf{F} = (x + yz)\mathbf{i} + (y - xz)\mathbf{j} + (z - e^x \sin y)\mathbf{k}$$

out of (D) through

(a) the whole surface (S),

(b) the surface (S_1),

(c) the surface (S_2).

(6) Evaluate

$$\iint_{(S)} \textbf{curl } \mathbf{F} \cdot \hat{\mathbf{N}} dS$$

where (S) is the surface $x^2 + y^2 + 2(z-1)^2 = 6, z \geq 0$, $\hat{\mathbf{N}}$ is the unit outward (away from the origin) normal on (S) and

$$\mathbf{F} = (xz - y^3 \cos z)\mathbf{i} + x^3 e^z \mathbf{j} + xyze^{x^2+y^2+z^2}\mathbf{k}.$$

(7) Solve the following heat conduction equation by the method of separation of variables:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad (0 < x < \pi, t > 0)$$

$$u(0,t) = u(\pi,t) = 0, \quad (t > 0)$$

$$u(x,0) = f(x), \quad (0 \leq x \leq \pi)$$

assuming that

(a) $f(x) = 2\sin 3x - \sin 5x$;

(b) $f(x) = \begin{cases} x & 0 \leq x \leq \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} \leq x \leq \pi. \end{cases}$

(8) Use Fourier series to solve the following wave equation:

$$\frac{\partial^2 u}{\partial t^2} - 4\frac{\partial^2 u}{\partial x^2} = 0, \quad (0 < x < 1, t > 0)$$

$$u_x(0,t) = u_x(1,t) = 0, \quad (t > 0)$$

$$u(x,0) = \cos^2 \pi x, \quad u_t(x,0) = \sin^2 \pi x \cos \pi x.$$