MCGILL UNIVERSITY

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATH 263

ORDINARY DIFFERENTIAL EQUATIONS AND LINEAR ALGEBRA

Examiner: Sidney Trudeau
Associate Examiner: Wilbur Jonsson

Date: Friday Dec. 7th, 2007
Time: 2:00pm - 5:00pm

INSTRUCTIONS

1. Do all work in space provided. Should more space be needed, an additional examination booklet will be provided.
2. Calculators are not allowed.
3. This is a closed book examination.
4. No dictionaries allowed, except for translation dictionaries.
5. 1 mark will be given for properly entering your name and id number in the space provided.
6. Marks on this paper add up to 100, and will be pro-rated combined with the term marks to calculate the final grade.
7. This exam is comprised of the cover page and 13 pages.

Name:__________________________ Student Number: __________

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1. (5 marks) Solve $y' = \dfrac{x - e^{-x}}{y + e^{y}}$.

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2. (5 marks) Solve $y dx + (2xy - e^{-2y}) dy = 0$

3. (5 marks) Solve $y' = \frac{x^2 + 3y^2}{2xy}$ Hint: Observe that the function $F(x, y) = \frac{x^2 + 3y^2}{2xy}$ is homogeneous.

4. (10 marks) Given that $y = e^x$ is a solution of $(x-1)y'' - xy' + y = 0$, find a second linearly independent solution to the differential equation.

5. (10 marks) Solve $y^{(iv)} - 2y'' + y = xe^x$

6. (15 marks) Solve $y'' + y = cos^2x$ using variation of parameters.

7. (15 marks) Using Laplace transforms, solve

$$y'' + 2y' + 2y = cos(t) + \delta(t - \pi/2)$$

with initial conditions $y(0) = 0, y'(0) = 0$.

8. Consider the linear transformation $T_A : R^3 \longrightarrow R^3$ defined by

$$T_A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \text{where } A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

relative to the standard basis.

(a) (4 marks) Find a basis for ker(A).

(b) (2 marks) Find a basis for the rowspace of A.

(c) (2 marks) Find a basis for the column space of A.

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9. (i) (4 marks) Determine the eigenvalues of

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

(ii) (3 marks) As well, determine bases for the corresponding eigenspaces.

(iii) (2 marks) Hence display an ordered basis of eigenvectors.

(iv) (2 marks) Finally, display the matrix of the linear transformation associated with A relative to the ordered basis of eigenvectors supplied in part (iii).

For the continuation of question 9.

10. (15 marks) Using matrix methods (eigenvalues, etc) find the general solution of the following non-homogeneous system

$$\mathbf{X}' = \begin{pmatrix} 2 & 8 \\ 0 & 4 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 2 \\ 16t \end{pmatrix} \quad \text{where } \mathbf{X} = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\dfrac{1}{s}, \quad s > 0$
2. e^{at}	$\dfrac{1}{s-a}, \quad s > a$
3. t^n, $n =$ positive integer	$\dfrac{n!}{s^{n+1}}, \quad s > 0$
4. t^p, $p > -1$	$\dfrac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5. $\sin at$	$\dfrac{a}{s^2+a^2}, \quad s > 0$
6. $\cos at$	$\dfrac{s}{s^2+a^2}, \quad s > 0$
7. $\sinh at$	$\dfrac{a}{s^2-a^2}, \quad s > \lvert a\rvert$
8. $\cosh at$	$\dfrac{s}{s^2-a^2}, \quad s > \lvert a\rvert$
9. $e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}, \quad s > a$
10. $e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}, \quad s > a$
11. $t^n e^{at}$, $n =$ positive integer	$\dfrac{n!}{(s-a)^{n+1}}, \quad s > a$
12. $u_c(t)$	$\dfrac{e^{-cs}}{s}, \quad s > 0$
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$
14. $e^{ct}f(t)$	$F(s-c)$
15. $f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \quad c > 0$
16. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
19. $(-t)^n f(t)$	$F^{(n)}(s)$