

MCGILL UNIVERSITY

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATH 263

ORDINARY DIFFERENTIAL EQUATIONS AND LINEAR ALGEBRA

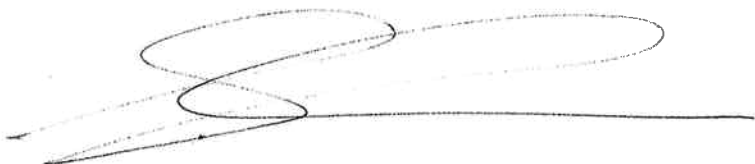
Examiner: Sidney Trudeau
Associate Examiner: Wilbur Jonsson

Date: Friday Dec. 7th, 2007
Time: 2:00pm - 5:00pm

INSTRUCTIONS

1. Do all work in space provided. Should more space be needed, an additional examination booklet will be provided.
2. Calculators are not allowed.
3. This is a closed book examination.
4. No dictionaries allowed, except for translation dictionaries.
5. 1 mark will be given for properly entering your name and id number in the space provided.
6. Marks on this paper add up to 100, and will be pro-rated combined with the term marks to calculate the final grade.
7. This exam is comprised of the cover page and 13 pages.

Name: _____ Student Number: _____

A handwritten signature in black ink, consisting of a large, stylized 'S' shape with a horizontal line extending to the right.

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No calculators. Do all work in space provided.

1. (5 marks) Solve $y' = \frac{x - e^{-x}}{y + e^y}$.

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2. (5 marks) Solve $ydx + (2xy - e^{-2y})dy = 0$

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3. (5 marks) Solve $y' = \frac{x^2 + 3y^2}{2xy}$ Hint: Observe that the function $F(x, y) = \frac{x^2 + 3y^2}{2xy}$ is homogeneous.

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4. (10 marks) Given that $y = e^x$ is a solution of $(x - 1)y'' - xy' + y = 0$, find a second linearly independent solution to the differential equation.

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5. (10 marks) Solve $y^{(iv)} - 2y'' + y = xe^x$

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6. (15 marks) Solve $y'' + y = \cos^2 x$ using variation of parameters.

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7. (15 marks) Using Laplace transforms, solve

$$y'' + 2y' + 2y = \cos(t) + \delta(t - \pi/2)$$

with initial conditions $y(0) = 0, y'(0) = 0$.

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8. Consider the linear transformation $T_A : R^3 \rightarrow R^3$ defined by

$$T_A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ where } A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

relative to the standard basis.

- (a) (4 marks) Find a basis for $\ker(A)$.
- (b) (2 marks) Find a basis for the row space of A .
- (c) (2 marks) Find a basis for the column space of A .

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9. (i) (4 marks) Determine the eigenvalues of

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

(ii) (3 marks) As well, determine bases for the corresponding eigenspaces.

(iii) (2 marks) Hence display an ordered basis of eigenvectors.

(iv) (2 marks) Finally, display the matrix of the linear transformation associated with A relative to the ordered basis of eigenvectors supplied in part (iii).

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For the continuation of question 9.

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10. (15 marks) Using matrix methods (eigenvalues, etc) find the general solution of the following non-homogeneous system

$$\mathbf{X}' = \begin{pmatrix} 2 & 8 \\ 0 & 4 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 2 \\ 16t \end{pmatrix} \text{ where } \mathbf{X} = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

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TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}, \quad s > 0$
2. e^{at}	$\frac{1}{s-a}, \quad s > a$
3. $t^n, n = \text{positive integer}$	$\frac{n!}{s^{n+1}}, \quad s > 0$
4. $t^p, p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5. $\sin at$	$\frac{a}{s^2 + a^2}, \quad s > 0$
6. $\cos at$	$\frac{s}{s^2 + a^2}, \quad s > 0$
7. $\sinh at$	$\frac{a}{s^2 - a^2}, \quad s > a $
8. $\cosh at$	$\frac{s}{s^2 - a^2}, \quad s > a $
9. $e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}, \quad s > a$
10. $e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}, \quad s > a$
11. $t^n e^{at}, n = \text{positive integer}$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$
14. $e^{ct}f(t)$	$F(s-c)$
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$
16. $\int_0^t f(t-\tau)g(\tau) d\tau$	$F(s)G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$
19. $(-t)^n f(t)$	$F^{(n)}(s)$