FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS MATH 248

Honours Advanced Calculus

Examiner: Professor S. W. Drury | Date: Thursday, 17 December 2009

Associate Examiner: Professor A. Hundemer | Time: 9: 00 am. – 12: 00 noon.

INSTRUCTIONS

Answer all questions in the booklets provided.
This is a closed book examination.
Non-programmable calculators only are permitted.
Both regular and translation dictionaries are allowed.

Read the questions carefully before answering them.
Write your solutions in a clear, complete and logical way.
Simplify all answers as far as possible.
Do not use the calculator to replace exact expressions by approximations.

This exam has 7 questions and 2 pages

1. (10 points) Find the absolute maximum and absolute minimum values taken by the function $f(x, y, z) = 9x^2 + 12y^2 + 2z^3$ on the set $\{(x, y, z); x^2 + y^2 + z^2 \leq 25, z \geq 0\}$. Justify that these absolute extrema exist.

2. (10 points) The transformation $f(x, y, z) = (u, v, w)$ given by $u = x + \frac{1}{2}y^2$, $v = y + \frac{1}{2}z^2$, $w = z + \frac{1}{2}x^2$ is one-to-one on the cube $S = \{(x, y, z); 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1\}$. Find the volume and centre of mass of the region $f(S)$ in uvw-space. In calculating the centre of mass be sure to assume a uniform distribution of mass relative to the (u, v, w) coordinates.

3. (10 points) Consider the transformation $f(x, y, z) = (u, v, w)$ given by $u = x+\frac{1}{2}y^2$, $v = y+\frac{1}{2}z^2$, $w = z+\frac{1}{2}x^2$. Note that $f(1, 0, 0) = (1, 0, \frac{1}{2})$. Find the tangent vector to the curve $t \mapsto f^{-1}(1-t, t, \frac{1}{2})$ at $t = 0$. What theorem guarantees that f^{-1} is differentiable at $(1, 0, \frac{1}{2})$? Explain why the theorem is applicable in this situation.

4. (10 points) Find the volume of the region of $\mathbb{R}^3$ defined by the inequalities $\sqrt{x^2 + y^2} \leq z \leq 1 - \max(|x|, |y|)$ (the intersection of a cone and a pyramid).

5. (10 points) Find the surface area of the portion of the surface in $\mathbb{R}^3$ parametrized by $(s, t) \mapsto (9s^2, 12st, 8t^2)$ corresponding to the range $0 \leq s \leq 1$, $0 \leq t \leq 1$ of the parameters. Find also the area of the region in the xy-plane obtained by projecting this portion of surface parallel to the z-axis onto the xy-plane.

6. (10 points) Use the Divergence Theorem to compute the integral $\iint_{\partial R} \vec{F} \cdot d\vec{S}$ representing the flux of the vector field $\vec{F}(x, y, z) = 3x^2\vec{\imath} + (-2xy + y)\vec{\jmath} + 5z\vec{k}$ out of the region R defined by the inequalities $|x| + |y| \leq 1$, $|y| + |z| \leq 1$ and $|z| + |x| \leq 1$. Note: Be sure to exploit the symmetry of R to eliminate odd terms in the integrand.

7. (10 points) Use Stokes' Theorem to compute the integral $\int_{\partial S} \vec{F} \cdot d\vec{s}$ where $\vec{F}(x, y, z) = 3x^2\vec{\imath} + (-2xy + y)\vec{\jmath} + 5z\vec{k}$ where S is the portion of the cone $z = \sqrt{x^2 + y^2}$ satisfying the inequalities $x \geq 0, y \geq 0, z \leq 1$ oriented in the direction of increasing z.

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