**STUDENT NAME:** ______________________________

**STUDENT NUMBER:** ________________

**FACULTY OF SCIENCE**

**FINAL EXAMINATION**

**MATHEMATICS 189-222A**

**CALCULUS III**

Examiner: W. Jonsson — Date: ????????, December ???, 2010
Associate Examiner: N. Sancho — Time: 9:00 AM - 12:00 PM

**Instructions**

1. Total number of points: 100.
2. No books, calculators or notes are allowed for the exam. Do not rip pages from the examination book.
3. There are 4 versions of this examination. **This version belongs to Group 1**.
4. Answers to Part I are to be entered on the machine readable sheet with a soft lead pencil.
5. Answers to part II are to be written in the space provided on the examination paper.
6. Your answers may contain $\pi$ or other expressions that cannot be computed without a calculator, e.g. $\ln 2$, $300^{1/2} + 13 \cdot 150^{-3/2}$.
7. All material (question papers, machine readable sheets) must be turned in.
8. Name, Student number and group number of your examination **MUST** be entered on the question paper and on the machine readable sheet.

**GOOD LUCK!**

**Score Table**

| | |
|---|---|
| Part I Multiple Choice | |
| Part II Problems | Points |
| 1. | |
| 2. | |
| 3. | |
| 4. | |
| 5. | |
| **Total:** | |

This exam comprises 7 pages, including this cover.

**PART I. Multiple choice questions.** **Group 1**

Each question is worth 3 points.

1. The equation of the plane tangent to the surface $z = 2x^2 + y^2$ at the point where $x = 3$, $y = 2$ is

   (a) $z = -12x - 4y + 66$, (b) $z = 12x - 4y - 6$, (c) $z = -12x + 4y + 50$,

   (d) $z = -12x + 4y - 30$, (e) $z = 12x + 4y - 22$.

2. The fourth degree Taylor polynomial of $f(x) = x^2 e^x$ centered at $a = 0$ is

   (a) $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$, (b) $1 + x + \frac{x^2}{2} + \frac{x^3}{6}$, (c) $x + x^2 + \frac{x^3}{2} + \frac{x^4}{6}$,

   (d) $x^2 + x^3 + \frac{x^4}{2}$, (e) $\frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24}$.

3. The vector $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ is perpendicular to the vector $9\mathbf{i} - 3\mathbf{j} + c \cdot \mathbf{k}$ when

   (a) $c = -1$, (b) $c = 1$, (c) $c = 2$, (d) $c = 3$, (e) $c = 0$ .

4. For any two vectors $\mathbf{a}, \mathbf{b}$ the cross product $(3\mathbf{a} + 2\mathbf{b}) \times \mathbf{a}$ is the same vector as

   (a) $2\mathbf{a} \times \mathbf{b}$, (b) $2\mathbf{b} \times \mathbf{a}$, (c) $3\mathbf{a} \times \mathbf{b}$, (d) $3\mathbf{b} \times \mathbf{a}$, (e) $5\mathbf{b} \times \mathbf{a}$.

5. The series $\sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^n$

   (a) has sum 1/2, (b) has sum 1/3, (c) has sum 2/3,

   (d) diverges to $\infty$, (e) diverges to $-\infty$.

6. The $p$-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$

   (a) converges for $p \geq 1$ and diverges for $p < 1$,

   (b) converges for $p > 1$, diverges for $p < 1$ and we cannot say whether is converges or diverges for $p = 1$,

   (c) diverges for $p \geq 1$ and converges for $p < 1$,

   (d) diverges for $p > 1$ and converges for $p \leq 1$,

   (e) converges for $p > 1$ and diverges for $p \leq 1$.

7. The power series $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$ represents the function

   (a) $e^x$, (b) $\sin x$, (c) $\cos x$, (d) $\arctan x$, (e) $\tan x$.

**Group 1**

8. The power series $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(n+1)!2^{2n+1}}$

   (a) converges only for $x = 0$, (b) has radius of convergence 2,

   (c) converges for all real numbers $x$, (d) has interval of convergence $-4 < x < 4$,

   (e) has radius of convergence 1.

9. A particle is moving along the trajectory $\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 3\sin t\,\mathbf{j} + t\,\mathbf{k}$. At time $t = \pi/2$ the velocity vector $\mathbf{v}(\pi/2)$ and the acceleration vector $\mathbf{a}(\pi/2)$ are

   (a) $\mathbf{v}(\pi/2) = -2\mathbf{i} + \mathbf{k}$, and $\mathbf{a}(\pi/2) = -3\mathbf{j}$,
   (b) $\mathbf{v}(\pi/2) = -2\mathbf{j} + \mathbf{k}$, and $\mathbf{a}(\pi/2) = -3\mathbf{i}$,
   (c) $\mathbf{v}(\pi/2) = 2\mathbf{i} + \mathbf{k}$, and $\mathbf{a}(\pi/2) = 3\mathbf{j}$,
   (d) $\mathbf{v}(\pi/2) = 2\mathbf{j} + \mathbf{k}$, and $\mathbf{a}(\pi/2) = 3\mathbf{i}$,
   (e) $\mathbf{v}(\pi/2) = 3\mathbf{i} + \mathbf{k}$, and $\mathbf{a}(\pi/2) = 2\mathbf{j}$.

10. The directional derivative of the function $f(x, y) = x^2 + 2y^2$ in the direction of the unit vector $\mathbf{u} = (\mathbf{i} - \mathbf{j})/\sqrt{2}$ and at the point $(2, 3)$ is

    (a) 0, (b) $2/\sqrt{2}$, (c) $-4/\sqrt{2}$, (d) $6/\sqrt{2}$, (e) $-8/\sqrt{2}$.

11. The series expansion of $\int e^{x^2}\,dx$ is

    (a) $1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots,$

    (b) $x + x^3 + \frac{x^5}{2!} + \frac{x^7}{3!} + \frac{x^9}{4!} + \cdots,$

    (c) $x + \frac{x^3}{3} + \frac{x^5}{10} + \frac{x^7}{3!7} + \frac{x^9}{4!9} + \cdots,$

    (d) $x + \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \frac{x^9}{9} + \cdots,$

    (e) $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5} + \cdots.$

12. At the point $(2, 1)$ the direction in which the function $f(x, y) = \frac{x^2}{4} + y^2$ has the maximum rate of change is given by the vector

    (a) $-\mathbf{i} - 2\mathbf{j}$, (b) $-\mathbf{i} + 2\mathbf{j}$, (c) $2\mathbf{i} + \mathbf{j}$, (d) $\mathbf{i} + 2\mathbf{j}$, (e) $\mathbf{i} - 2\mathbf{j}$.

**Group 1**

13. The equation of the plane tangent to the surface $e^x + 2y + \sin z = 0$ at the point $(0, -1/2, 0)$ is

    (a) $x + 2y + z = -1$, (b) $x - 2y + z = 1$, (c) $2x + y + z = -1/2$,

    (d) $2x - y + z = -1/2$, (e) $x + 2y - z = -1$.

14. The tangent plane to the level surface $x^2 + \frac{y^2}{4} + z^2 = 3$ of the function $F(x, y, z) = x^2 + \frac{y^2}{4} + z^2$ at the point $(1, 2, 1)$ has equation

    (a) $-2x + y + 2z = 2$, (b) $2x - y + 2z = 2$, (c) $2x - y - 2z = -2$,

    (d) $2x + y + 2z = 6$, (e) $2x + y - 2z = 2$.

**END OF PART I**

**PART II.**

1. (15 points) Let $f(x,y)$ be a differentiable function and $x = 2s\cos t$, $y = 3s\sin t$.

(a) Find expressions for $\frac{\partial f}{\partial s}$, $\frac{\partial f}{\partial t}$ in terms of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(b) Show that, for the same change of variables, we have

$$4\left(\frac{\partial f}{\partial x}\right)^2 + 9\left(\frac{\partial f}{\partial y}\right)^2 = \left(\frac{\partial f}{\partial s}\right)^2 + \frac{1}{s^2}\left(\frac{\partial f}{\partial t}\right)^2.$$

2. (16 points) Consider the function $f(x, y) = 2x^2 + 8xy + y^4$.

(a) Find the critical points and classify them as local maxima, local minima and saddle points.

(b) For the same function approximate $f(1.99, 1.02)$ with the help of differentials.

3. (9 points) Change the order of integration and evaluate the following integral

$$\int_0^1 \int_{x^2}^1 x^3 \sin(y^3)\, dy\, dx.$$

4. (8 points) Find the volume under the cone $z = \sqrt{x^2 + y^2}$ and above the region in the $xy$-plane lying between the two circles $x^2 + y^2 = 1$, $x^2 + y^2 = 4$. Use polar coordinates.

5. (10 points) Find the extreme values of the function

$$f(x, y) = 2x + y$$

on the ellipse

$$\frac{x^2}{2} + \frac{y^2}{8} = 1.$$

Which are maxima and which are minima?

**END of PART II**

**Continuation of solution for problem: ______.**
**You must refer to this page on the page where the problem is printed.**

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