McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATH 222

CALCULUS 3

Examiner: Mohammad Alakhrass
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Date: Friday April 27, 2007
Time: 9:00 AM - 12:00PM

INSTRUCTIONS

1. Please attempt to answer all questions in the exam booklets provided.
2. This is a closed book exam. No notes or books are permitted.
3. Calculators are not permitted.
4. Use of a regular and or translation dictionary are permitted.

This examination is comprised of the cover page and, 2 pages of 6 questions.

Q1 Let $f(x) = \dfrac{4x^3}{4+x^4}$.

(a) (5 marks) Write the Maclaurin series for the function f and find the radius of convergence.

(b) (4 marks) Write the Maclaurin series for the function $g(x) = \ln(4+x^4)$. (Use part (a))

(c) (4 marks) Find $f^{(2007)}(0)$. (Don't simplify your answer).

Q2 Consider the curve with position vector $\mathbf{r}(t) = (8t, 6\sin(t), 6\cos(t))$.

(a) (6 marks) Find the Frenet frame $\hat{T}(t), \hat{N}(t), \hat{B}(t)$ for the curve $\mathbf{r}$.

(b) (6 marks) Find the curvature, the radius of curvature and the center of curvature for $\mathbf{r}$.

(c) (3 marks) Find the arc length of the curve $\mathbf{r}$ from the point $\mathbf{r}(0)$ to the point $\mathbf{r}(2\pi)$.

(d) (3 marks) Reparametrize $\mathbf{r}(t)$ in terms of the arc length s.

(e) (3 marks) Find equations of the tangent line to the curve $\mathbf{r}$ at the point where $t = \dfrac{\pi}{2}$.

(f) (4 marks) If a particle moves with position vector $\mathbf{r}(t)$ find the tangential and normal components of its acceleration.

Q3 **(a)** (6 marks) Show that the function

$$f(x) = \begin{cases} \frac{ax+y^2}{\sqrt{x^2+y^2}} & \text{If } (x,y) \neq (0,0) \\ 0 & \text{If } (x,y) = (0,0) \end{cases}$$

is continuous at $(0,0)$ if $a = 0$, but is not continuous if $a \neq 0$.

(b) (5 marks) Let f and g be twice differentiable functions of one variable. Define $u(x,t) = f(x+2t) + g(x-2t)$.
Compute $\dfrac{\partial^2 u}{\partial t^2} - 4\dfrac{\partial^2 u}{\partial x^2}$.

Q4 Let $f(x,y,z) = y^4 + xy^3 + x^2yz + z^2$, $\hat{u} = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$, $p = (1,1,1)$.

(a) (3 marks) Find the rate of change of f at p in the direction $\hat{u}$ (i.e. $D_{\hat{u}}f(p)$).

(b) (3 marks) In what direction at the point p does f increase most rapidly? Find the rate of increase in that direction.

(c) (3 marks) Find an equation of the tangent plane to the surface $f(x,y,z) = 4$ at the point p.

Q5 **(a)** (6 marks)Find and classify the critical points of the function $f(x,y) = xy(1-10x-2y)$ as local maxima, local minima or saddle points.

(b) (6 marks) Using Lagrange multipliers find the the maximum and the minimum values of $f(x,y) = x^2 + y^2$ on the circle $x^2 + y^2 - 4x + 3 = 0$.

Q6 **(a)** (6 marks) Evaluate $\int_0^1 (\int_{x^2}^1 x^3 \sin(y^3)\, dy)\, dx$.

(b) (6 marks) Evaluate $\int_0^\infty e^{-x^2/2}\, dx$.

(c) (6 marks) Evaluate $\int\int\int_R z\, dV$, where R is the region between the two cylinders $x^2 + y^2 = 1$, $x^2 + y^2 = 4$ bounded below by the plane $z = 0$, and bounded above by the paraboloid $z = x^2 + y^2$.

(d) (6 marks) Find the volume of the region D , where D is the region that lies **inside** the sphere $x^2 + y^2 + z^2 = 9$ and **outside** the cone $z = -\sqrt{x^2 + y^2}$.

(e) (6 marks) Calculate the surface area of the part of the surface $z = x^2 + y^2 - 2x + 1$ over the disk $(x-1)^2 + y^2 \leq 1$.