<u>McGILL UNIVERSITY</u>

<u>FACULTY OF SCIENCE</u>

<u>FINAL EXAMINATION</u>

<u>MATH 222-001</u>

<u>CALCULUS 3</u>

Examiner: Professor J. Toth
Associate Examiner: Professor Klemes

Date: Tuesday April 18, 2006
Time: 9:00 am - 12:00 pm

<u>INSTRUCTIONS</u>

1. Answer questions in the exam booklets provided.
2. Please show all your work. Each question is worth 10 points.
3. This is a closed book exam.
4. Calculators are not permitted.
5. Use of a regular and or translation dictionary is not permitted.

This exam comprises of the cover page, and 2 pages of 7 questions.

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MATH 222 CALCULUS 3: Final Examination

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1. Test the following series for convergence:

$$(a)\ \sum_{n=1}^{\infty} \frac{2\cos(n^2)}{n^2},$$

$$(b)\ \sum_{n=1}^{\infty} (-1)^n n \sin\frac{1}{\sqrt{n}},$$

$$(c)\ \sum_{n=1}^{\infty} e^{-n} \sin n.$$

2. (a) Write down the Maclaurin series for

$$f(x) = \frac{2x}{1+x^2}.$$

(b) What is the interval of convergence of the series in part (a)?
(c) Using the series in part (a), write down the Maclaurin series for $g(x) = \ln(1+x^2)$. What is the interval of convergence?
(d) Compute $\ln(1.5)$ to within an accuracy of 0.05.

3. Given $f(x,y,z) = x^2 + y^2 + 3z^4$, consider the level surface $f(x,y,z) = 4$.
(a) What is the maximal rate of increase of $f(x,y,z)$ at the point $P = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1\right)$ on the level surface $f(x,y,z) = 4$?
(b) What is the equation of the tangent plane to $f(x,y,z) = 4$ at $P = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1\right)$?
(c) Write down the parametric equations of the normal line to $f(x,y,z) = 4$ at $P = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1\right)$.

4. Use the method of Lagrange multipliers to find the global maximum and global minimum of the function $f(x,y) = x^2 + 3y$ on the sphere $x^2 + y^2 + z^2 = 1$.

5. (a)Showing all your work, determine and classify all critical points of

$$f(x,y) = x\,e^{-x^3+y^3}$$

as local maxima, local minima or saddle points.
(b) Compute the global extrema of f over the disk $x^2 + y^2 \le 1$.

6. Let $z = f(x, y)$ be a twice continuously differentiable function satisfying the *Laplace equation:*

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0.$$

Let θ be fixed real number and define $w = f(x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta)$. Compute

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}.$$

7. (a) Evaluate

$$\int\int_{x^2+y^2\leq 1} (\sin x + y^3 + 3)dA.$$

(b) Evaluate

$$\int\int_R xy^2 dA,$$

where R is the finite region in the first quadrant bounded by the curves $y = x^2$ and $x = y^2$.