NOTE TO PRINTER

Cover pg only on blue

(These instructions are for the printer. They should not be duplicated.)

THIS EXAMINATION SHOULD BE PRINTED ON $8\frac{1}{2} \times 14$ PAPER, AND STAPLED WITH 3 SIDE STAPLES, SO THAT IT OPENS LIKE A LONG BOOK.

There are FOUR versions of this examination. Each of them should be printed with a DIFFERENT COLOURED COVER. IN THE EXAMINATION ROOM VERSIONS ##1,2 OF THE BOOKS SHOULD BE ALTERNATED IN ONE ROW (=STRIPE=COLUMN) AND VERSIONS ##3,4 IN THE ROWS (=STRIPES=COLUMNS) ON EITHER SIDE, WITH A DIFFERENT PAIR OF BOOKS USED IN ALTERNATE ROWS, SO THAT NO STUDENT IS NEXT ON ANY SIDE TO A STUDENT WRITING THE SAME COLOUR OF EXAMINATION.

NORMALLY THIS BOOK IS ENOUGH FOR ALL THE STUDENT'S WRITTEN WORK, including rough work; STUDENTS SHOULD <u>NOT</u> NORMALLY BE GIVEN A BLANK EXAMINATION BOOKLET IN ADDITION.

VERSION 1

McGILL UNIVERSITY
FACULTY OF SCIENCE
FINAL EXAMINATION

MATHEMATICS 222 2005 09 — CALCULUS 3

EXAMINER: Professor W. G. Brown
ASSOCIATE EXAMINER: Prof. P. Kassaei

DATE: Monday, December 19th, 2005
TIME: 14:00 – 17:00 hours

FAMILY NAME:

GIVEN NAMES:

STUDENT NUMBER:

INSTRUCTIONS

1. Fill in the above clearly.
2. DO NOT TEAR PAGES FROM THIS BOOK! All your writing — even rough work — must be handed in. You may do rough work anywhere in the booklet.
3. Calculators are not permitted.
4. The examination booklet consists of this cover, Pages 1 through 7 containing questions; and Pages 8, 9, and 10, which are blank. *Your neighbour's version of this examination may not be the same as yours.*
5. There are two kinds of problems on this examination, each clearly marked as to its type.
   - Some of the questions on this paper require that you SHOW ALL YOUR WORK! Their solutions are to be written in the space provided on the page where the question is printed. When that space is exhausted, you may write *on the facing page*. Any solution may be continued on the last pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed!
   - Some of the questions on this paper require only BRIEF SOLUTIONS ; for these you are expected to write the correct answer in the box provided; you are not asked to show your work, and you should not expect partial marks for solutions that are not completely correct.

   You are expected to simplify your answers wherever possible.

   You are advised to spend the first few minutes scanning the problems. (Please inform the invigilator if you find that your booklet is defective.)
6. A TOTAL OF 70 MARKS ARE AVAILABLE ON THIS EXAMINATION.

PLEASE DO NOT WRITE INSIDE THIS BOX

1(a) /3	1(b) /4	1(c) /3	2(a) /2	2(b) /4	2(c) /4	3(a) /6	3(b) /4	4(a) /2
4(b) /2	4(c) /2	4(d) /4	5 /10	6(a) /5	6(b) /5	7 /10		
								TOTAL /70

1. BRIEF SOLUTIONS [10 MARKS]

Consider the function $f(x, y, z) = x - \frac{y^2}{8} - \frac{z^2}{18}$.

(a) [3 MARKS] Give an equation for the level surface of f passing through the point $(4, 2, 3)$.

ANSWER ONLY

(b) [4 MARKS] Give an equation for the tangent plane through $P(4, 2, 3)$ to the level surface of f passing through P.

ANSWER ONLY

(c) [3 MARKS] At the point $(4, 2, 3)$ determine the rate of change of $f(x, y, z)$ with respect to distance in the direction of $2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

ANSWER ONLY

2. [10 MARKS]

(a) BRIEF SOLUTIONS [2 MARKS] For the function $\frac{1}{1+x}$, the Maclaurin series is

ANSWER ONLY

The interval of convergence of this series is

ANSWER ONLY

(b) SHOW ALL YOUR WORK! [4 MARKS] *Use your result in the preceding part* to determine the Maclaurin series for $f(x) = \ln(1-x)$. Justify all steps in your derivation, and *state without proof* the interval of convergence of the series you obtain.

(c) SHOW ALL YOUR WORK! [4 MARKS] When $x = \frac{1}{2}$, carefully determine a partial sum of the Maclaurin series for f that could be used to approximate $\ln \frac{1}{2}$ to within an error of 0.01.

3. SHOW ALL YOUR WORK! [10 MARKS]

(a) [6 MARKS] For the curve

$$\mathbf{r}(t) = \left(\frac{2}{1+t^2} - 1\right)\mathbf{i} + \frac{2t}{1+t^2}\mathbf{j} - 2\mathbf{k} \qquad (-\infty < t < +\infty)$$

determine — simplified as much as possible — a formula for the distance $s(t)$ from the point with parameter value $t = -1$ measured in the direction of increasing t. Show all your work.

(b) [4 MARKS] Showing all your work, determine — simplified as much as possible — parametric equations for the tangent to the curve at the point where $t = 2$.

4. [10 MARKS] Let $I = \int_0^2 \int_y^{\sqrt{8-y^2}} e^{-x^2-y^2}\,dx\,dy\,.$

(a) BRIEF SOLUTIONS [2 MARKS] Sketch the region over which this iterated integral is taken.

ANSWER ONLY

(b) BRIEF SOLUTIONS [2 MARKS] Write I as one or more iterated integrals with the order of integration reversed from that of the given iteration.

ANSWER ONLY

(c) BRIEF SOLUTIONS [2 MARKS] Write I as an iterated integral in polar coordinates.

ANSWER ONLY

(d) SHOW ALL YOUR WORK! [4 MARKS] Evaluate I.

5. SHOW ALL YOUR WORK! [10 MARKS] Use the method of Lagrange multipliers — no other method will be accepted for this problem — to find all points on the surface $x^2+10y^2+z^2=5$ where the function $8x-4z$ has a global maximum, and all points where it has a global minimum.

6. SHOW ALL YOUR WORK! [10 MARKS]

(a) [5 MARKS] Showing all your work, determine and classify all critical points of the function

$$f(x, y) = 3xy - x^2y - xy^2$$

as local maxima, local minima, or saddle points.

(b) [5 MARKS] Determine the global extrema of f over the triangle with vertices at $(0, 0)$, $(0, 4)$, and $(4, 0)$.

7. SHOW ALL YOUR WORK! [10 MARKS] Suppose that $z = f(u, v)$, where f has continuous second partial derivatives. If $u = e^s \cos t$, and $v = e^s \sin t$, carefully determine the value of

$$(u^2 + v^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right) - \left(\frac{\partial^2 z}{\partial s^2} + \frac{\partial^2 z}{\partial t^2}\right)$$

and show that it is constant.

CONTINUATION PAGE FOR PROBLEM NUMBER

You *must* refer to this continuation page on the page where the problem is printed!

CONTINUATION PAGE FOR PROBLEM NUMBER ☐

You *must* refer to this continuation page on the page where the problem is printed!

CONTINUATION PAGE FOR PROBLEM NUMBER ☐

You *must* refer to this continuation page on the page where the problem is printed!