

Version 1

McGill University
Faculty of Science

Final Examination

Mathematics 141 FALL 09

Calculus 2

Date: Friday, Dec. 11th 2009

Time: 14:00 – 17:00 hours

EXAMINER: Prof. N. Sancho

ASSOCIATE EXAMINER: Prof. W. Brown

FAMILY NAME:
OTHER NAME:
STUDENT NUMBER:

INSTRUCTIONS

1. Calculators are not permitted
2. You can continue your work in the **preceding (or facing) page** or in the pages at the end of the booklet.
3. This examination booklet consists of a cover page, plus 14 pages; pages 11, 12, 13 and 14 are blank. You must not tear pages from this booklet.
4. Dictionaries are not allowed
5. This is a closed book examination.

1(a)	1(b)	1(c)	2	3(a)	3(b)	4(a)	4(b)	5	6(a)
6(b)	7(a)	7(b)	8(a)	8(b)	8(c)	Total			

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1. SHOW ALL YOUR WORK

(a) [2 marks] If $F(x) = \int_0^{\sqrt{x}} \frac{t^2}{1+t^4} dt$ find the derivative of $F(x)$.

(b) [3 marks] Let $F(x) = 2x + \int_x^0 \frac{\sin 2t}{1+t^4} dt$. Determine $F(0)$; $F'(0)$; $F''(0)$.

(c) [5 marks] Sketch the region bounded by the curves $y = 12 - x^2$ and $y = x^2 - 6$ and find its area.

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2. SHOW ALL YOUR WORK

[7 marks] Sketch the region bounded by the curves; $x = y^3$, $x = 8$, $y = 0$ and find the volume of the solid generated by revolving the region about the y-axis.

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3. SHOW ALL YOUR WORK

(a) [8 marks] Calculate $\int \frac{x+4}{x^2+2x+5} dx$.

(b) [7 marks] Calculate $\int (\ln x)^2 dx$.

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4. SHOW ALL YOUR WORK

(a) [8 marks] Prove the reduction formula for all integers $n \geq 2$

$$\int \sec^n x \, dx = \frac{\tan x \sec^{n-2} x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx.$$

(b) [7 marks] Evaluate $\int \sec^5 x \tan^2 x \, dx$.

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5. SHOW ALL YOUR WORK

[10 marks] Find the exact area of the surface obtained by rotating the curve about the x-axis: $x = t^3$, $y = t^2$, $0 \leq t \leq 1$.

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6. SHOW ALL YOUR WORK

(a) [10 marks] Evaluate $\int \frac{\sin^3(\ln x) \cos^3(\ln x)}{x} dx$

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6.SHOW ALL YOUR WORK

(b) [5 marks] Evaluate the improper integral $\int_2^3 \frac{dx}{\sqrt{3-x}}$.

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7. SHOW ALL YOUR WORK

(a) [5 MARKS] find the arc length of the curve from $x = 1$ to $x = 2$ of $y = x^2 - \frac{1}{8} \ln x$.

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7. SHOW ALL YOUR WORK

(b) [10 MARKS] Find the area of the inner loop of $r = 1 + 2\sin \theta$. (Hint: sketch the curve)

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8. SHOW ALL YOUR WORK

(a) [4 marks] Determine whether the given series converges or diverges $\sum_{n=1}^{\infty} \frac{n!}{n^2 e^n}$

(b) [3 marks] Determine whether the geometric series $\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$ is convergent or

divergent. If it is convergent find its sum.

(c) [6 marks] Determine whether the series converges absolutely, converges

conditionally or diverges: $\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{(n+1)\ln(n+1)}$

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CONTINUATION PAGE FOR PROBLEM NUMBER

You must refer to this continuation page on this page where the problem is printed.