

MCGILL UNIVERSITY  
FACULTY OF SCIENCE  
FINAL EXAMINATION

MATHEMATICS 140 2006 09

CALCULUS I

EXAMINER: Professor W. G. Brown  
ASSOCIATE EXAMINER: Prof. N. Sancho

DATE: Thursday, December 07th, 2006  
TIME: 09:00 – 12:00 hours

FAMILY NAME:

GIVEN NAMES:

STUDENT NUMBER:

INSTRUCTIONS

1. Fill in the above clearly.
2. DO NOT TEAR PAGES FROM THIS BOOK! All your writing — even rough work — must be handed in. You may do rough work anywhere in the booklet.
3. This is a CLOSED BOOK examination. CALCULATORS ARE NOT PERMITTED. Translation dictionaries are permitted; no other dictionaries are permitted.
4. OTHER CALCULUS EXAMINATIONS ARE BEING WRITTEN AT THIS TIME. THIS IS THE EXAMINATION IN MATH 140 ONLY!
5. The examination booklet consists of this cover, Pages 1 through 8 containing questions; and Pages 9, 10, and 11, which are blank. *Your neighbour's version may not be the same as yours.*
6. There are two kinds of problems on this examination, each clearly marked as to its type.
  - Some of the questions on this paper require that you SHOW ALL YOUR WORK! Their solutions are to be written in the space provided on the page where the question is printed. When that space is exhausted, you may write *on the facing page*. Any solution may be continued on the last pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed!
  - Some of the questions on this paper require only BRIEF SOLUTIONS; for these you are expected to write the correct answer in the box provided; you are not asked to show your work, and you should not expect partial marks for solutions that are not completely correct.

You are expected to simplify your answers wherever possible.

You are advised to spend the first few minutes scanning the problems. (Please inform the invigilator if you find that your booklet is defective.)

7. A TOTAL OF 70 MARKS ARE AVAILABLE ON THIS EXAMINATION.

PLEASE DO NOT WRITE INSIDE THIS BOX

1(a)	1(b)	1(c)	1(d)	1(e)	2(a)	2(b)	2(c)	2(d)
/2	/2	/2	/2	/2	/3	/3	/3	/3
3(a)	3(b)	3(c)	3(d)	4(a)	4(b)	4(c)	5	6(a)
/2	/2	/2	/2	/1	/4	/4	/8	/4
6(b)	7	8(a)	8(b)	8(c)	8(d)			TOTAL
/3	/8	/2	/2	/3	/1			/70

1. **BRIEF SOLUTIONS**

[2 MARKS EACH] Give the numeric value of each of the following limits if it exists; if the limit is  $+\infty$  or  $-\infty$ , write  $+\infty$  or  $-\infty$  respectively. In all other cases write “NO FINITE OR INFINITE LIMIT”.

(a)  $\lim_{x \rightarrow \infty} \frac{\sin 2x}{x} =$

ANSWER ONLY

(b)  $\lim_{x \rightarrow 0} \frac{\sin 2x}{x} =$

ANSWER ONLY

(c)  $\lim_{x \rightarrow \infty} \left(1 - \frac{2}{x} + \frac{7}{x^2}\right)^x =$

ANSWER ONLY

(d)  $\lim_{x \rightarrow 0} \frac{|x|}{x} =$

ANSWER ONLY

(e)  $\lim_{x \rightarrow -\infty} (\sqrt{x^2 + 3x} + x) =$

ANSWER ONLY

## 2. BRIEF SOLUTIONS

[3 MARKS EACH] For each of the following functions answer the question; if the object requested does not exist, write “NONE”.

- (a) The horizontal asymptotes to the graph of  $g(x) = 2 \tanh x + 1$  are

ANSWER ONLY

- (b) The vertical asymptotes to the graph of  $h(x) = \sqrt{|x^2 - 1|}$  are

ANSWER ONLY

- (c) A ladder 4 metres long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a speed of 1 metre per second, how fast is the angle between the top of the ladder and the wall changing when the angle is  $\frac{\pi}{6}$  radians?

ANSWER ONLY

- (d) The removable discontinuities of  $m(x) = \begin{cases} x^2 & \text{if } x < -1 \\ 0 & \text{if } -1 \leq x \leq 0 \\ 2x & \text{if } 0 < x < 3 \\ 2x & \text{if } 3 < x \end{cases}$

are at the following points:

ANSWER ONLY

## 3. BRIEF SOLUTIONS

[2 MARKS EACH] Evaluate each of the following, and always simplify your answers as much as possible.

(a)  $\frac{d}{dx} \left( \frac{x^{\frac{1}{2}} + 3x^{\frac{2}{3}}}{x} \right) =$

ANSWER ONLY

(b)  $\frac{d}{du} e^{\ln(\ln u)} =$

ANSWER ONLY

(c) An antiderivative  $F(x)$  of  $f(x) = \frac{1}{1+x^2}$  such that  $F(0) = 3$  is

ANSWER ONLY

(d)  $\frac{d}{dt} \left( t \arcsin(t) + \sqrt{1-t^2} \right) =$

ANSWER ONLY

4. SHOW ALL YOUR WORK!

- (a) [1 MARKS] For a real number  $a$  in the domain of a function  $f$ , define precisely what is meant by  $f'(a)$ .
- (b) [4 MARKS] Let  $f(x) = |1 - x|$ . Carefully, using the preceding definition, either determine  $f'(1)$ , or prove that  $f$  is not differentiable at  $x = 1$ . (No marks will be given for a solution that uses the Differentiation Rules.)
- (c) [4 MARKS] It is known that  $x$  and  $y$  are related by the equation  $\sqrt{x + y} = 1 + x^2y^2$ . Using any valid method, determine  $\frac{dy}{dx}$  when  $x = 0$ .

5. **SHOW ALL YOUR WORK!**

[8 MARKS] Showing all your work, determine constants  $a, b, c$  so that the tangent lines at  $x = -1$  and  $x = 5$  to the graph of

$$y = f(x) = ax^2 + bx + c$$

have slopes 6 and  $-2$  respectively, and that the graph passes through the point  $(1, 4)$ .

6. **SHOW ALL YOUR WORK!**

A particle is moving in the plane on the curve  $C$  whose equation is

$$x = 2y - 4 + e^y.$$

- (a) [4 MARKS] Use the Mean Value Theorem to prove that  $C$  crosses the  $y$ -axis exactly once. (You may assume it is known that  $2 < e < 3$ .)
- (b) [3 MARKS] If  $P(0, a)$  is the point where  $C$  crosses the  $y$ -axis, give a linear approximation in terms of  $a$  for the  $y$ -coordinate of the point on  $C$  where the  $x$ -coordinate is  $\frac{1}{2}$ .

7. **SHOW ALL YOUR WORK!**

[8 MARKS] Showing all your work, use the calculus to find two positive real numbers such that the sum of the first and the square of the second is 300, and the product of the two numbers is as large as possible.



8. **SHOW ALL YOUR WORK!**

Let  $f(x) = x - \ln(1 + x^2)$ .

- (a) [2 MARKS] Showing all your work, determine the intervals where  $f$  is increasing, and the intervals where it is decreasing.
- (b) [2 MARKS] Showing all your work, determine whether  $f$  has local extrema, and classify them, if any, as maxima or minima.
- (c) [3 MARKS] Showing all your work, determine all inflection points for  $f$ .
- (d) [1 MARK] Sketch the graph of  $f$ .

CONTINUATION PAGE FOR PROBLEM NUMBER

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