

NOTE TO PRINTER

(These instructions are for the printer. They should not be duplicated.)

**THIS EXAMINATION SHOULD BE PRINTED ON  $8\frac{1}{2} \times 14$   
PAPER, AND STAPLED WITH 3 SIDE STAPLES, SO THAT IT  
OPENS LIKE A LONG BOOK.**

MCGILL UNIVERSITY  
FACULTY OF SCIENCE  
FINAL EXAMINATION

MATHEMATICS 189-140A

CALCULUS I

EXAMINER: Professor K. K. Tam

DATE: December 11, 1998

ASSOCIATE EXAMINER: Professor W. G. Brown

TIME: 9:00 – 12:00 hours

SURNAME:

MR, MISS, MS, MRS, &c.:

GIVEN NAMES:

STUDENT NUMBER:

INSTRUCTIONS

1. Fill in the above clearly.
2. Do not tear pages from this book; all your writing — even rough work — must be handed in.
3. Calculators are not permitted.
4. This examination booklet consists of this cover, Pages 1 through 12 containing questions; and Pages 13 and 14, which are blank. You are expected to show all your work. All solutions are to be written in the space provided on the page where the question is printed. When that space is exhausted, you may write *on the facing page*. Any solution may be continued on the last pages, or the back cover of the booklet, but you must indicate any continuation clearly on the page where the question is printed!
5. You are advised to spend the first few minutes scanning the problems. (Please inform the invigilator if you find that your booklet is defective.)

PLEASE DO NOT WRITE INSIDE THIS BOX

1	2	3(a)	3(b)	4	5(a)	5(b)	5(c)	6(a)
/4	/5	/4	/4	/8	/3	/3	/2	/3
6(b)	7	8	9	10	11(a)	11(b)	11(c)	11(d)
/3	/12	/6	/8	/10	/2	/4	/2	/2
11(e)	12(a)	12(b)	12	14	TOTAL	TOTAL		

1. [4 MARKS] Showing your work, find  $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x - 3}$ .

2. [5 MARKS] Given that a function  $f$  has the property that

$$|f(x) - 2| \leq (x - 1)^2,$$

determine  $\lim_{x \rightarrow 1} f(x)$ .

3. (a) [4 MARKS] Let  $v(x) = x^3 - 4x^2 + x + 3$ . Use the Intermediate Value Theorem to show that the equation  $v(x) = 0$  has a solution between  $x = 1$  and  $x = 2$ .
- (b) [4 MARKS] By examining the behavior of  $v(x)$  as  $x \rightarrow \infty$  and as  $x \rightarrow -\infty$ , or otherwise, discuss the existence of other solutions to  $v(x) = 0$ .

4. [8 MARKS] Find equations for all straight lines which are both normal to the curve  $y = \sqrt{x-3}$  and parallel to the straight line  $y = -2x + 11$ .

5. For the function  $g(x) = |1 - x^2|$ ,
- (a) [3 MARKS] Sketch the graph of  $g$ .
  - (b) [3 MARKS] Show that  $g$  is continuous at  $x = 1$ .
  - (c) [2 MARKS] Determine whether  $g$  is differentiable at  $x = 1$ .

6. (a) [3 MARKS] Determine  $h'(x)$ , if  $h(x) = \frac{\sin x}{1 - 2 \cos x}$ .

(b) [3 MARKS] If  $u(x) = \ln((2x - 1)^3)$ , determine  $u'(x)$ .

7. [12 MARKS] Determine the slope of the curve

$$(x + y)^2 - (x - y)^2 = x^4 + y^4$$

at all point(s), other than the origin, where the curve meets the line  $x = y$ .

8. [6 MARKS] A child is building a snowman by rolling a snowball on the ground; its volume is increasing at the rate of 8 cubic centimetres per minute. Find the rate at which the radius is increasing when the snowball is 75 centimetres in diameter. (The volume of a sphere of radius  $r$  is  $\frac{4}{3}\pi r^3$ .)

9. [8 MARKS] The domain of the function  $F$  is to be taken to be the interval  $[-2, \frac{1}{2}]$ . On that interval,

$$F(x) = x^3 + x^2 - x + 1.$$

Determine the global maximum (= absolute maximum), global minimum (= absolute minimum), local maxima (= relative maxima), and local minima (= relative minima).

10. [10 MARKS] A closed box with a square base is to have a volume of 2,000 cubic centimetres. The material for the top and bottom of the box costs \$3 per square centimetre, while the material for the sides costs \$1.50 per square centimetre. Determine the dimensions of the least expensive box.

11. For the function  $f(x) = \frac{x^2}{x^2 - 4}$ ,

- (a) [2 MARKS] Determine the (largest possible) domain.
- (b) [4 MARKS] Determine all local extrema, and all points of inflection.
- (c) [2 MARKS] Determine precisely where the function is increasing, decreasing, concave upward, concave downward.
- (d) [2 MARKS] Determine all horizontal and all vertical asymptotes, if any.
- (e) [1 MARK] Sketch the graph.

12. It is given that the function  $f$ , defined by  $f(x) = x + x^3$  has an inverse, denoted by  $f^{-1}$ .

(a) [3 MARKS] Determine the value of  $f^{-1}(2)$ .

(b) [3 MARKS] Determine the value of  $\left(\frac{d}{dx}(f^{-1})\right)(2)$ .

13. [4 MARKS] Determine the derivative of the function

$$m(x) = x^x.$$

14. [4 MARKS] Determine the value of the following limit, if it exists:

$$\lim_{x \rightarrow 0} \frac{e^{x^3} - 1}{x - \sin x}$$

CONTINUATION PAGE FOR PROBLEM NUMBER

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