

1. (15 marks) Compute the following limits.

$$(a) \lim_{x \rightarrow 0} \frac{\tan(2x)}{\sin(7x)}, \quad (b) \lim_{h \rightarrow 0^+} \frac{\sqrt{h}}{\sqrt{4 + \sqrt{h}} - 2}, \quad (c) \lim_{x \rightarrow 1} \frac{x^3 - 1}{x^2 - 1}.$$

2. (a) (15 marks) Differentiate the following functions.

i) $f(x) = (\ln(x^2 + 1))^{\frac{1}{3}} + e^{-x^3+x},$

ii) $f(x) = \frac{3x + 7}{4^x}.$

- (b) Compute $\frac{dy}{dx}.$

$$x^2 \sin y = x^3 + y^3$$

3. (a) (15 marks) Find the equation of the tangent line to the curve $y = f(x)$ at the point (4,-6), where

$$f(x) = (1 - x)\sqrt{x}.$$

- (b) Find the equation of the tangent line to the curve defined by the equation

$$2e^{-x} + e^y = 3e^{x-y}$$

at the point (0,0).

- (c) The volume of a spherical balloon is being increased at the rate $3\text{cm}^3/\text{sec}$. How fast are the radius r and the surface area S increasing when $r = 13\text{cm}$?

4. (30 marks) Sketch the graph of

$$f(x) = x + 2 + \frac{4}{x - 2}$$

indicating where the function is increasing, decreasing, concave up, concave down, has local maxima, local minima, inflection points, and asymptotes (if they exist).

5. (a) (15 marks) Show that the function

$$f(x) = \begin{cases} (x - 1)^3 & \text{if } x \leq 1 \\ 2x - 2 & \text{if } x > 1 \end{cases}$$

is injective in $(-\infty, \infty)$ and find its inverse. Is $f(x)$ continuous at 1? Is $f(x)$ differentiable at 1? Explain.

- (b) Show that the equation $1 - x = \sin x$ has exactly one solution in the interval $\left(0, \frac{\pi}{2}\right)$.

6. (10 marks) A farmer has 1600 meters of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

McGILL UNIVERSITY
FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-139B

CALCULUS

Examiner: R. Trujillo-Cortez
Associate Examiner: Professor O. Kharlampovich

Date: Wednesday, April 21, 1999
Time: 2:00 P.M. - 5:00 P.M.

NOTE: Another calculus examination is being written at the same time in this building. Please ensure that this is the examination in YOUR course.

INSTRUCTIONS

1. Clearly enter your family name, given name, and student identification number in the examination booklet.
2. The use of books, calculators, or notes is **NOT** permitted for the Final examination.
3. Do not tear pages from the examination booklet. All written work, including rough work, is to be handed in. Rough work can be done on the back of each page. Circle the final answer. There are 6 questions. You must answer **ALL SIX** questions. **Complete (justified) answers are required.**
4. This examination consists of 2 pages of questions plus this cover page. Verify that your exam is not defective; if it is defective, please inform the invigilator.