

1. (10%) Evaluate the limits

$$(a) \lim_{x \rightarrow 2} \left( \frac{1}{x-2} - \frac{4}{x^2-4} \right), \quad (b) \lim_{x \rightarrow 0} \frac{x \sin x}{x^2 + 2x + 1}.$$

2. (15%) Differentiate the functions

$$(a) y = \frac{\sin(x^2 + 1)}{x^2 + 2}$$

$$(b) y = \ln \left( \frac{1}{\sqrt{x^2 + 1}} \right) \sin^{-1}(x)$$

$$(c) y = xe^{-x} \ln(x + 2).$$

3. (30%) Sketch the graph of  $f(x) = \frac{x^2 + x - 2}{x^2}$ , showing maxima, minima, inflection points and asymptotes.

4. (20%) A window has perimeter 10m and is in the shape of a rectangle with top edge replaced by a semicircle. Find the dimensions of the rectangle if the window admits the greatest amount of light (i.e. so the window has maximum area). Do not approximate  $\pi$ .

5. (10%) Find the equation of the line tangent to the curve  $xe^y + y - 2x = \ln 2$  at the point  $(1, \ln 2)$ .

6. (10%) Show that the equation  $x^7 + 5x^3 + 2x - 6 = 0$  has exactly one real root.

7. (5%) Find a constant  $a$  that will make the function  $f$  continuous at  $x = 1$  if

$$f(x) = \begin{cases} \frac{x^2 - 3x^2 + 2}{x^2 - 1} & \text{for } x \neq 1 \\ a & \text{for } x = 1. \end{cases}$$

McGILL UNIVERSITY  
FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-139A

CALCULUS

Examiner: Professor O. Kharlampovich  
Associate Examiner: Professor W.G. Brown

Date: Friday, December 11, 1998  
Time: 9:00 A.M. - 12:00 Noon.

INSTRUCTIONS

**NO CALCULATORS ARE PERMITTED.  
ANSWER ALL QUESTIONS.**

This exam comprises the cover and 1 page of questions.