1. Evaluate, showing all details,
$$\int_{-\infty}^{\infty} \frac{\sin(x)}{x(x^2+1)} dx \ .$$

2. Find the residues of :
   (a) $\dfrac{e^z}{z^2(1+z^2)}$ at all singularities,
   (b) $\dfrac{e^{1/z}}{1-z}$ at $z = 0$.

3. Let $f(z)$ be an entire function.
   (a) Prove that for all $R > 0$,
$$|f^{(n)}(0)| \le \frac{n!}{R^n} \text{Max}\{|f(z)|: \quad |z| = R \quad \}.$$
   (b) If $|f(z)| \le A + B|z|^{7/2}$ for some constants A and B, prove that f is a polynomial of degree ≤ 3.

4. (a) Define isolated singularity of $f(z)$.
   (b) Define removable singularity of $f(z)$ and pole of $f(z)$.
   (c) If a is an isolated singularity of $f(z)$ and if
$$\lim_{z \to a} (z-a) f(z) = 0,$$
   prove that a is removable.
   (d) If a is an isolated singularity of $f(z)$ and if $|f(z)| \to \infty$ when $z \to a$, prove that a is a pole of f.

5. Find a $1-1$ analytic mapping of the upper half plane $U = \{z\colon \quad \mathrm{Im} z > 0\}$ onto the strip $S = \{z\colon \quad 0 < \mathrm{Re} z < 1\}$.

6. Find the number of zeroes of $e^z + 4iz$ in $|z| \leq 1$. Explain your work.

7. Let $f(z)$ be analytic on the closed upper half plane $D = \{z\colon \quad \mathrm{Im} z \geq 0\}$, and suppose that $|f(z)| \to 0$ as $|z| \to \infty$ in D. Show that

$$\sup_{z \epsilon D} |f(z)| \quad = \quad \sup_{x \epsilon \mathbf{R}} |f(x)|.$$

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-466A

COMPLEX ANALYSIS

Examiner: Professor I. Klemes
Associate Examiner: Professor K.P. Russell

Date: Friday, December 11, 1998
Time: 2:00 pm - 5:00 pm

INSTRUCTIONS

NO CALCULATORS PERMITTED
Show all work and simplify answers.
Answer all 7 questions.
Keep this exam paper.

This exam comprises the cover and 2 pages of questions.