1. A furniture manufacturer produces chairs and sofas, which are sold at prices of \$160 and \$140, respectively. The manufacturing process requires the following person-hours of labor per item:

	Carpentry	Upholstery	Finishing
Sofa	6	2	1
Chair	3	6	1

Each week there are at most 240 hours of carpentry time, 180 hours of upholstery time, and 45 hours of finishing time available.

   (a) How many sofas and how many chairs should the manufacturer produce each week to maximize the profit? Formulate the problem as a linear program and solve it by the simplex method.

   (b) An extra person-hour of carpentry, upholstery and finishing costs the manufacturer \$10, \$15, and \$30, respectively. Should the manufacturer buy the extra person-hour and, if so, in which manufacturing process, in order to increase the profit?

   (c) The manufacturer has two options to improve the efficiency (by purchasing new machines). He can either decrease the number of person-hours required in the carpentry process to make a chair or he can decrease the number of required person-hours in the upholstery process to make a sofa. Which option should he choose?

2. Consider the feasible set F in $\mathbb{R}^4$ determined by the constraints:

$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 10 \\ x_1 - 2x_2 + 2x_3 + 4x_4 &= 16 \\ x_i \geq 0, \quad i &= 1, 2, 3, 4. \end{aligned}$$

   (a) Find all extreme points of the set F.

   (b) Identify an extreme point x^* of F that is closest to the origin. (Use the Euclidean norm.)

   (c) Is the extreme point x^*, identified in (b), a point in F closest to the origin? Answer this question using the Karush-Kuhn-Tucker conditions.

3. Consider the problem

$$\begin{aligned} \text{Min} \quad f(x) = \ & x_1^2 + x_2^2 + x_3^2 + x_4^2 \\ \text{s.t.} \quad & \\ & x_1 + x_2 + x_3 + x_4 = 10 \\ & x_1 - 2x_2 + 2x_3 + 4x_4 = 16 \\ & x_i \geq 0, \quad i = 1, 2, 3, 4. \end{aligned}$$

(a) Verify that

$$x^0 = \begin{bmatrix} 4 \\ 0 \\ 6 \\ 0 \end{bmatrix}$$

is not an optimal solution.

(b) Using the Cauchy method of steepest descent (modified for the problems with constraints) find a better feasible point x^1. Verify that $f(x^1) < f(x^0)$.

4. Consider the program

$$\begin{array}{llllll} \text{Max} & x_1 & +x_4 & & & \\ \text{s.t.} & & & & & \\ & x_1 & +x_2 & +x_3 & +x_4 & = 2 \\ & x_1 & -2x_2 & +2x_3 & +4x_4 & = 3 \\ & x_i & \geq 0, & i = 1, 2, & 3, 4. & \end{array}$$

(a) Using Farkas' lemma, verify that the feasible set is not empty.

(b) Write the corresponding dual problem. Solve the dual using the graphic approach.

(c) Using only the optimal value of the dual found in (ii), solve the above (primal) program. Do <u>not</u> use the simplex method!

5. The following table gives data for six fast food restaurants:

	Staff Hours	Supplies	Profit	Customers
A	400	120	100	1050
B	360	100	90	1200
C	390	130	105	1100
D	410	125	110	1000
E	280	80	60	900
F	300	90	70	850

Consider Staff Hours and Supplies as input variables and Profit and Customers as output variables.

(a) Formulate (but do not solve) the Charnes-Cooper-Rhodes efficiency test for the restaurant F.

(b) It has been found that the optimal weights for the restaurant F are $x_1^* = 0.0033$, $x_2^* = 0$, $y_1^* = 0.0091$, $y_2^* = 0.0003$ (after normalization), the efficiency ratio is $q^* = 0.8920$, and shadow prices are $p_A^* = 0$, $p_B^* = 0.454$, $p_C^* = 0.278$, $p_D^* = 0$, $p_E^* = 0$, $p_F^* = 0$.

Using this information determine the Charnes-Cooper-Rhodes projection on the efficiency frontier. In particular, determine the inputs and outputs that will make the restaurant F efficiently administered.

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-417A/487A

MATHEMATICAL PROGRAMMING

Examiner: Professor S. Zlobec — Date: Friday, December 19, 1997
Associate Examiner: Professor N.G.F. Sancho — Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

Attempt all problems.

Calculators are permitted.

This exam comprises the cover and 2 pages of questions.