MARKS

1. Solve the following ordinary differential equations:

(5) (a) $y' + 2y = e^{-x}$.

(5) (b) $(4x^3y^{-2} + 3y^{-1})dx + (3xy^{-2} + 4y)dy = 0$.

(5) (c) $(3xy + y^2) + (x^2 + xy)y' = 0$.

(5) (d) $x^2y'' + xy' + (x^2 - 1/4)y = 3x^{3/2}\sin x (x > 0)$, given that $y_1(x) = x^{-1/2}\sin x$ and $y_2(x) = x^{-1/2}\cos x$ solve the associated homogeneous equation.

(15) 2. (a) Let

$$f(x) = \begin{cases} x; & 0 \leq x < 1 \\ f(x-1); & x \geq 1. \end{cases}$$

Compute $\mathcal{L}\{f\}(s)$.

(b) Let

$$f(t) = \begin{cases} 0; & t < 1 \\ t^2 - 2t + 2; & t \geq 1. \end{cases}$$

Compute $\mathcal{L}\{f\}(s)$.

(c) Compute $\mathcal{L}^{-1}\left(\frac{1 - e^{-2s}}{s^2}\right)$.

(d) Compute $\mathcal{L}^{-1}(F(as + b))$ in terms of $\mathcal{L}^{-1}(F(s))$. Justify your answer.

(20) 3. (a) Solve

$$y'' + 2y' + y = \delta(t) + u_{2\pi}(t)$$
$$y(0) = 0,\ y'(0) = 1\ .$$

(b) Find the solution of the initial value problem:

$$y'' + 4y = g(t)$$
$$y(0) = 3,\ y'(0) = -1\ .$$

Express your answer as a convolution integral.

(25) 4. (a) Solve in a series centered at $x = 0$:

$$(3 - x^2)y'' - 3xy' - y = 0 \ .$$

What is the radius of convergence?

(b) Solve in a series centered at $x = 0$:

$$(1 - x)y'' + y = 0 \ .$$

(15) 5. Solve the heat equation

$$u_t = u_{xx}$$

on the strip $t \geq 0, \ -1 \leq x \leq 1$, with boundary conditions

$$u(-1, t) = u(1, t) = 0$$
$$u(x, 0) = 7\sin(4\pi x) \ .$$

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-325B

ORDINARY DIFFERENTIAL EQUATIONS

Examiner: Professor J. Toth — Date: Thursday, April 17, 1997

Associate Examiner: Professor K.P. Russell — Time: 2:00 P.M. - 5:00 P.M.

This exam comprises the cover and 2 pages of questions.