1. Solve the following initial value problems.

   (i) (10 marks) $\frac{dy}{dx} = y^2 \cos x, \qquad y(0) = 1.$

   (ii) (10 marks) $y\frac{dy}{dx} = 3x^2 + 2x^3 - y^2, \qquad y(0) = 1.$

2. (i) (10 marks) Solve the initial value problem

$$\frac{d^2y}{dx^2} = y^2\left(\frac{dy}{dx}\right)^3, \qquad y(0) = 1, \frac{dy}{dx}(0) = 1,$$

   leaving your answer in implicit from.

   (ii) (10 marks) Find the general solution of the equation

$$x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = 0.$$

3. (20 marks) Given that $y_1(x) = \sec x$ and $y_2(x) = \tan x$ are solutions of the equation

$$\frac{d^2y}{dx^2} - (\tan x)\frac{dy}{dx} - (\sec^2 x)y = 0,$$

   find the general solution of the equation

$$\frac{d^2y}{dx^2} - (\tan x)\frac{dy}{dx} - (\sec^2 x)y = 1.$$

4. Use the *Method of Undetermined Coefficients* to find the solution of the initial value problem

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 4xe^x, \qquad y(0) = 0, \frac{dy}{dx}(0) = 1.$$

5. (20 marks) Find a fundamental set of solutions of the equation

$$x^2(1 - x^2)\frac{d^2y}{dx^2} - 2y = 0$$

   expressed as series in powers of x.

6. Suppose that a function g is defined by

$$g(t) = \begin{cases} 0 & \text{if } t < 0 \\ t & \text{if } 0 \le t \le 1 \\ 1 & \text{if } t > 1 \end{cases}$$

(i) (5 marks) Find the Laplace transform $\mathcal{L}g$ of g.

(ii) (10 marks) Find the Laplace transform $\mathcal{L}y$ of the solution y of the initial value problem

$$y'' - 2y' + y = g(t), \qquad y(0) = 0, y'(0) = 0.$$

(iii) (5 marks) Find the solution y itself.

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