1. Find the general solution of each of the equations,

   (a) $x^2 \dfrac{d^2y}{dx^2} = 4\left(\dfrac{dy}{dx}\right)^2$.

   (b) $x^2 \dfrac{d^2y}{dx^2} - 5x\dfrac{dy}{dx} + 9y = 0$.

   (c) $y''' + 2y'' - y' - 2y = 0$.

2. Find the complete general solution of

$$y'' + 9y = 2\sec 3x$$

   using variation of parameters. Note that $\int \tan v dv = \ln|\sec v| + c$.

3. (a) Find the solution (implicit) of the differential equation

$$(6xy - y^3)dx + (4y + 3x^2 - 3xy^2)dy = 0.$$

   (b) A perfectly flexible cable hangs over a frictionless peg with 8ft of cable on one side of the peg, 12ft on the other side, and the motion is governed by

$$\frac{d^2x(t)}{dt^2} - \frac{g}{10}x(t) = \frac{g}{5}.$$

   where g is acceleration due to gravity and $x(t)$ is the displacement of the cable in feet after t seconds. At $t = 0$, $x(0) = 0$ and the cable is at rest. Solve the initial value problem for $x(t)$. You may leave g a constant in your answer. Find the time it takes for 8ft of cable to move over the peg.

4. Use the Method of Undetermined Coefficients to find the solution of the initial value problem

$$y'' + y' - 6y = 4te^{2t},\ y(0) = 0,\ y'(0) = 1.$$

5. Suppose that $g(t)$ is defined as follows,

$$g(t) = \begin{cases} 0 & t < 0 \\ t & 0 \le t < 1 \\ 0 & t \ge 1 \end{cases}$$

   (a) Find the Laplace transform of $g(t)$.

   (b) Solve the initial value problem

$$\begin{aligned} y'' - y' - 2y &= g(t) \\ y(0) = 0,\ y'(0) &= 0 \end{aligned}$$

   using Laplace transforms, giving

   i. the Laplace transform of the solution $y(t)$

   ii. and the solution $y(t)$ itself on the entire real line.

6. Show that the origin is a regular singular point of the equation,

$$2x^2y'' + (x^2 - x)y' + y = 0$$

and then find the general solution of this differential equation in terms of series. Find the radius of convergence of each of the solutions.

7. Consider the problem $y' = 1 - t + 4y,\ y(0) = 1$, using the Euler formula and step size $h = 0.1$, determine an approximate value of the solution $y = \phi(t)$ at $t = 0.2$ for this initial value problem.

McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-315A

ORDINARY DIFFERENTIAL EQUATIONS

Examiner: Dr. P. Bracken
Associate Examiner: Professor S.W. Drury

Date: Tuesday, December 15, 1998
Time: 9:00 A.M. - 12:00 Noon.

INSTRUCTIONS

This is a CLOSED book examination. Write your answers in the booklets provided.
A table of Laplace transforms is attached.

This exam comprises the cover, 2 pages of questions and 1 page of Laplace transforms.