1. Solve the following differential equations, with initial conditions, if given.

a) (5 points) $(y^2 cos(x) - 3x^2 y - 2x)dx + (2y\ sin(x) - x^3 + ln(y))dy = 0,\ y(0) = e.$

b) (5 points) $$6xydx + (4y + 9x^2)dy = 0$$

c) (5 points) $$y'' - 2y' + y = e^x/(1 + x^2)$$

d) (5 points) $$Y' = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \cdot Y$$

e) (5 points) $(3x + 1)y'' - (9x + 6)y' + 9y = 0$, given that $y_1 = e^{3x}$ is a solution

f) (5 points) $$y'' + 4y = 3sin(2x),\ y(0) = 2, y'(0) = -1$$

g) (5 points) $$x^2 y'' - 4xy' + 4y = 0$$

2. a) (35 points) Solve in a series centered at $x = 0$. You must first decide whether to use a regular series or a Frobenius series.

   i) $(1 - x^2)y'' - xy' + \alpha^2 y = 0$, where α is a constant.

   ii) $2xy'' + y' + xy = 0$.

   Discuss the radius of convergence of the solutions. In case a) for what values of the constant α are polynomial solutions obtained?

   b) For the equation $(x + 2)^2(x - 1)y'' + 3(x - 1)y' - 2(x + 2)y = 0$, say which points on the real line are regular, and which are regular singular.

3. a) (10 points) Compute the inverse Laplace transform of

$$\text{i)}\ \frac{(s-2)e^{-s}}{s^2-4s+3}, \qquad \text{ii)}\ \frac{1}{s^3(s+1)}.$$

b) (5 points) Compute the Laplace transform of the square wave function f, defined by $f(t) = 1$, if t lies in an interval $[2n, 2n+1)$, and $f(t) = -1$ if t lies in an interval $[2n+1, 2n+2)$, n an integer.

c) (15 points) Solve the initial value problem

$$y'' + y = u_{\pi/2}(t) + \delta(t-\pi) - u_{3\pi/2}(t),\ \ y(0) = 0, y'(0) = 0.$$

Solve the same problem, but with the initial conditions $y(0) = 1, y'(0) = 3$.