MCGILL UNIVERSITY

STUDENT NAME STUDENT #

Faculty of Science
FINAL EXAMINATION

MATHEMATICS- MATH 315
ORDINARY DIFFERENTIAL EQUATIONS

Examiner: Dr. L. Laayouni
Associate examiner: Prof. Dimitry Jakobson

Wednesday, December 15, 2004
14:00-17:00

INSTRUCTIONS

1. The use of books, notes are not allowed for this exam.
2. This exam paper may not be removed from the exam room by the student.
3. Non-programmable calculators are allowed.
4. This exam paper comprises the cover paper, one page of questions and one page containing the table of elementary Laplace transforms.
5. No Dictionaries of Any Kind

Question: 1
Show that the differential equation

$$(3x + \frac{6}{y}) + (\frac{x^2}{y} + 3\frac{y}{x})\frac{dy}{dx} = 0,$$

is not exact and that the function $\mu = xy$ is an integrating factor of this differential equation, then solve it using this integrating factor.

Question: 2
Find the general solution to the initial value problem

$$y'' + 2y' + 2y = e^{-x}\cos(x), \qquad y(0) = 1, \quad \text{and} \quad y'(0) = 1.$$

Question: 3
Show that the differential equation

$$(x-1)^2 y'' + 2(x^2 - 1)y' + 2y = 0, \qquad x > 1,$$

has a regular singular point at $x = 1$, and find the corresponding indicial equation, then discuss the nature of the two solutions near $x = 1$.

Question: 4
Use the Laplace transform to determine the solution of the initial value problem

$$y'' + 2y' + 2y = g(t), \qquad y(0) = 0, \quad y'(0) = 1,$$

where the forcing function $g(t)$ is given by

$$g(t) = \begin{cases} t, & t < 3\pi \\ 1, & t \geq 3\pi. \end{cases}$$

Question: 5
Use the improved Euler method with uniform step size $h = 0.1$, to calculate the values of the solution of the initial value problem

$$y' = 2 - t, \qquad y(0) = 1$$

at the points $t = 0.1$, $t = 0.2$, $t = 0.3$, $t = 0.4$, and $t = 0.5$.

Elementary Laplace Transforms	
$f(t)$	$\mathcal{L}(f)$
1	$\frac{1}{s}, \quad s > 0$
e^{at}	$\frac{1}{s-a}, \quad s > a$
t^n	$\frac{n!}{s^{n+1}}, \quad s > 0 \ \ (n = 0, 1, \ldots)$
$\sin at$	$\frac{a}{s^2 + a^2}, \quad s > 0$
$\cos at$	$\frac{s}{s^2 + a^2}, \quad s > 0$
$e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}, \quad s > a$
$e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}, \quad s > a$
$\sinh at$	$\frac{a}{s^2 - a^2}, \quad s > \|a\|$
$\cosh at$	$\frac{s}{s^2 - a^2}, \quad s > \|a\|$
$u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
$u_c(t) f(t-c)$	$e^{-cs} F(s), \quad s > 0$
$e^{ct} f(t)$	$F(s-c)$
$f''(t)$	$s^2 \mathcal{L}(f) - s f(0) - f'(0)$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - \cdots f^{(n-1)}(0)$