Final Examination December 11, 1996 Mathematics 189-261A

1. (a) Solve the initial-value problem

$$3y' + 6xy = 2x, \quad y(0) = 2.$$

(b) Find the solution of

$$3x^2 y dx + (3x^3 + 5y^2) dy = 0.$$

2. (a) Find two linearly independent solutions of

$$x^2 y'' - 3xy' + 4y = 0.$$

(b) Solve the initial-value problem

$$y'' = \frac{4}{3} y' y^3 \ ; \quad y(0) = 1 \text{ and } y'(0) = \frac{1}{3}.$$

3. Find the general solution in terms of real valued functions of $x$ for

$$y''' + 27y = 3x^2.$$

4. Consider the problem of finding series solutions in powers of $x$ for the Tchebycheff equation

$$(1 - x^2)y'' - xy' + \alpha^2 y = 0$$

where $\alpha^2$ is a constant.

(a) In the case $\alpha = 2$, find a polynomial solution satisfying the initial conditions $y(0) = 1$ and $y'(0) = 0$.
(b) When $\alpha$ is not an integer, the series expansions no longer terminate. In that case, for what values of $x$ would you expect the series to converge?

5. Following the procedure below, find a series solution in powers of $x$ for the differential equation

$$2x^2 y'' + xy' - (1 + 2x^2)y = 0.$$

Determine:
(a) the indicial equation and its roots;
(b) the recurrence formula;
(c) the general solution including the first three nonzero terms of each series.

6. Use a Laplace transform to solve the initial-value problem

$$y'' + 3y' + 2y = 1 - \delta(t - 2) + u_3(t), \quad y(0) = 0, \ y'(0) = 1.$$

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATHEMATICS 189-261A

DIFFERENTIAL EQUATIONS

Examiner: Professor S. Maslowe
Associate Examiner: Professor W. Jonsson

Date: Wednesday, December 11, 1996
Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

**Calculators Not Permitted**

This exam comprises the cover, 1 page of questions and 1 page of Laplace transforms.