1. (a) Test the following series for convergence by employing successively the comparison test and the integral test:

$$\sum_{n=2}^{\infty} \frac{1}{(n+1)(\ln n)^{3/2}} \ .$$

   (b) Determine the interval of convergence, including the end-points, of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^n}{2n+1} \ .$$

2. The series $\tan u = u + \dfrac{u^3}{3} + \dfrac{2u^5}{15} + \cdots$ converges to $\tan u$ for $-\pi/2 < u < \pi/2$. Use this result to solve (a) and (b) below.

   (a) Find the first three terms of the Maclaurin series for $\ln|\sec x|$. For what values of x should the series converge?

   (b) Find the first three nonzero terms of the Taylor series about $x = 1$ of the function $e^x \tan(x-1)$.

3. (a) Find $\vec{T}, \vec{N}$ and κ, the curvature, for the curve $\vec{r}(t) = \cos t\ \vec{\imath} + \sin t\ \vec{\jmath} + t\ \vec{k}$.

   (b) Calculate the arc length of the curve $\vec{r}(t) = \cosh t\ \vec{\imath} + \sinh t\ \vec{\jmath} + t\ \vec{k}$ for $0 \le t \le \ln 2$.

4. (a) If $u(x,y) = e^y f(y^2 - x^2)$, show that u satisfies the partial differential equation

$$y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = xu \ .$$

   (b) Let $E = f(P, V, T)$ be the internal energy of a gas that obeys the ideal gas law $PV = nRT$, where n and R are constants. Find $(\partial E/\partial T)_P$.

5. A downhill skier begins his run at the point $(9, 6, 730)$ of a mountain whose height is given by $z = 1000 - 2x^2 - 3y^2$. Given that he wishes to descend as rapidly as possible,

   (a) in which direction should he proceed initially, and;

   (b) what is the projection in the (x, y) plane of the path along which he should travel?

6. (a) Find the linearization of the function $z = f(x, y) = x^2 + 5xy - 2y^2$ at the point $(1, 2, 3)$.

   (b) Find and classify all critical points of the function

$$f(x, y) = 2x^3 - 4x^2 - y^2 + 2xy.$$

7. (a) By reversing the order of integration, evaluate

$$\int_0^9 \int_{\sqrt{y}}^3 \sin \pi x^3 \; dx dy.$$

   (b) Find the volume of the region bounded below by the plane $z = 0$, laterally by the cylinder $x^2 + y^2 = 1$, and above by the paraboloid $z = x^2 + y^2$.

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATHEMATICS 189-260B

INTERMEDIATE CALCULUS

Examiner: Professor S.A. Maslowe — Date: Friday, May 2, 1997
Associate Examiner: Professor N.G.F. Sancho — Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

Calculators Not Permitted

This exam comprises the cover and 2 pages of questions.