PART I: do any FOUR of the following SIX questions.

1. Test the following series for convergence (conditional or absolute) or divergence.

(a) $\sum_{n=1}^{\infty}(-1)^n \ln\left(\frac{n+1}{n}\right)$, (b) $\sum_{n=1}^{\infty}(-1)^n \frac{2n+1}{n^2(n+1)^2}$, (c) $\sum_{n=1}^{\infty}\sin\left(\frac{\pi}{2^n}\right)$.

2. (a) Find the interval of convergence of the power series $\sum_{n=0}^{\infty}(n+1)(x-2)^n$

(b) i. Knowing the series for $\sin x$, find the Maclaurin series (Taylor series about 0) for

$$f(x) = \begin{cases} \dfrac{\sin x}{x} & \text{if } \ x \neq 0 \\ 1 & \text{if } \ x = 0 \end{cases}$$

ii. Use this series to compute the value of

$$\int_0^{0.02} \frac{\sin x}{x}\, dx$$

with an error less than 5×10^{-5}. Justify your answer.

3. Consider the function $f(x) = e^{x^2+x}$.

(a) For the Taylor series of $f(x)$ about $x = 0$, find the Taylor polynomials $T_1(x)$ of degree one and $T_2(x)$ of degree two.

(b) Write down an expression for the remainder when $f(x)$ is approximated by $T_1(x)$, then use this formula to estimate the error when $f(0.3)$ is approximated by $T_1(0.3)$.

4. Find the directional derivative of the function

(a)
$$f(x, y, z) = x^2 + y^2 + xyz$$
at the point $P(1, 1, -1)$ in the direction of the vector $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$.

(b) Find the equation of the tangent plane to the surface $f(x, y, z) = 1$ at the point $P(1, 1, -1)$ with the function f being the same as in part (a).

5. The equations

$$\begin{aligned} u^2 + v^2 + xy - x + y &= 5 \\ vxy - y^2 &= 1 \end{aligned}$$

define x and y implicitly as functions of u and v.

Compute

$$\begin{pmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{pmatrix}$$

at the point $(x, y, u, v) = (1, 1, 0, 2)$.

6. (a) Let $H(p, q)$ be a function with continuous first partials and assume that $z = z(x, y)$ is determined implicitly by the relation $H(\frac{z}{x}, \frac{y}{x}) = 0$. Show that, for $x \neq 0$, $z \neq 0$

$\partial z \qquad \partial z$

PART II: Do any FOUR of the following FIVE questions

1. Find and classify the critical points of

$$f(x,y) = x^3 - y^3 + 3x^2 + 3y^2 - 9x$$

2. (a) Make a rough sketch of the curve given in plane polar co-ordinates by $r = 2\cos 2\theta$.
   (b) Find the equation of the tangent line to this curve at $\theta = \pi/8$.
   (c) Find the area of one loop of this curve.

3. (a) Find $\mathbf{T}$, $\mathbf{N}$ and κ as functions of t for the curve $\mathbf{r}(t) = \cos 2t\mathbf{i} + \sin 2t\mathbf{j} - 2t\mathbf{k}$.
   (b) Find the equation of the tangent line to this curve at $t = \frac{\pi}{4}$.
   (c) Compute the length of the arc for $0 \le t \le \frac{\pi}{4}$.

4. Find the volume V and the moment of inertia $I_{xx} = \int\int\int_V x^2 dV$ of the region bounded by the co-ordinate planes and the plane $x + y/2 + z/3 = 1$.

5. Find the area of the region enclosed in the first quadrant by the curves

$$4y = x^3, 9y = x^3, x = y^3, 9x = y^3,$$

which does not have $(0,0)$ on its boundary.

McGILL UNIVERSITY

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATHEMATICS 189-260B

INTERMEDIATE CALCULUS

Examiner: Professor W. Jonsson
Associate Examiner: Professor I. Klemes

Date: Wednesday, April 26, 2000
Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

Faculty Standard Calculators are permitted.
This exam is in TWO parts.
Part I: Do any FOUR of the 6 questions.
Part II: Do any FOUR of the 5 questions.
All questions are of equal weight.

Note: State clearly which questions you want marked, otherwise the first four questions which you attempt in each part will be the ones marked.

This exam comprises the cover and two pages of questions.