1. Find if the following series are divergent, conditionally convergent or absolutely convergent. (Justify your answers).

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{n^{\frac{3}{4}}} \; (-1)^n$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n . \sin(\frac{1}{n})}{n^{\frac{1}{4}}}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2 . (-1)^n}{e^{n^2}}$

2. If $Si(x) = \displaystyle\int_0^x \frac{\sin t}{t} dt$,

   (a) find a power series for $Si(x)$ about $x = 0$,

   (b) for what values of x does the series converge

   (c) find $Si(0.3)$ to 5 decimals. (Show the value of each term used and justify your accuracy.)

3. Find the radius of convergence and interval of convergence of the following series:

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n (2x-4)^{2n}}{16^n . n^2}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{9^n x^n}{(\ln(n+1))^n}$

4. For $t \geq 0$, the parametric representation of the curve C is

$$x = t \; , \quad y = 4t^{\frac{3}{2}}/3 \; , \quad z = t^2 .$$

   (a) Find the unit tangent, unit normal, and curvature at any point.

   (b) Find the arc length of C cut off between $z = 0$ in $z = 4$

   (c) If t is time, find the tangential and normal components of acceleration at any point.

5. (a) Find the critical (stationary) points

$$z = xy(4 - x - y)$$

(b) Classify the points as maximum, minimum or saddle points.

6. Sketch the region integration for the following integral

$$\int_{x=0}^{3} \int_{y=0}^{\sqrt{9-x^2}} (x^2 + y^2) \, . \, e^{(x^2+y^2)} dy \, . \, dx$$

Now evaluate the integral.

7. If a body is bounded by the cone $z = 6 - \sqrt{x^2 + y^2}$. and the paraboloid $z = x^2 + y^2$, with the density of the material being $\rho(x, y, z) = \sqrt{x^2 + y^2}$, find the mass of the body.

8. (a) Let $w = z^3 + 3z + 2x^2 + 3y^2$. Find ∇w at any point (x, y, z).

(b) For the surface $z^3 + 3z + 2x^2 + 3y^2 = 9$, show that $(1, 1, 1)$ lies on the surface and find the tangent plane and normal line to the surface at $(1, 1, 1)$.

(c) The equation in (b) defines $z = f(x, y)$ with $f(1, 1) = 1$. Find

$$f_x(x, y) = \frac{\partial z}{\partial x} f_y(x, y) = \frac{\partial z}{\partial y}$$

and then find $f_{xx}(1, 1)$, $f_{xy}(1, 1)$ and $f_{yy}(1, 1)$ and expand $f(x, y)$ as a Taylor series about $(1, 1)$ as far as the <u>second degree terms</u>.