Part I. Multiple choice questions **Group 1**

1. Which of the following matrices are elementary?

$$\text{(i)} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{(ii)} \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}, \quad \text{(iii)} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \text{(iv)} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

   (a) i, ii and iii,
   (b) iv and ii,
   (c) i and iii,
   (d) i, iii and iv,
   (e) ii and iv.

2. The distance between the point $P(6,3,3)$ and the line of vector parametric equation $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\lambda \in \mathbb{R}$, is

   (a) $\sqrt{\dfrac{13}{3}}$, (b) $\dfrac{7}{3}$, (c) $\sqrt{10}$, (d) 3, (e) $2\sqrt{7}$.

3. A is an $n \times n$ matrix whose cube is the matrix all of whose entries are zero (i.e. $A^3 = 0$). $I + A$ is invertible and its inverse is

   (a) $I - A$, (b) $I - A + A^2$, (c) $I + A + A^2$, (d) $I - A - A^2$, (e) $I + A - A^2$.

4. The planes $x+y+z=0$ and $x-y+z=0$ meet in a line L_1; the planes $x-y+z=2$ and $y=2$ meet in a line L_2. The lines L_1 and L_2

   (a) coincide;
   (b) do not meet and are not parallel;
   (c) do not meet and are parallel;
   (d) are orthogonal to each other and meet in a point;
   (e) meet in a point and are not orthogonal to each other.

5. Given $A = \begin{pmatrix} 2 & 4 & 6 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{pmatrix}$ the $(2,2)$ entry of its inverse A^{-1} is

   (a) $\dfrac{1}{3}$, (b) $-\dfrac{1}{3}$, (c) $\dfrac{1}{2}$, (d) $-\dfrac{1}{2}$, (e) 0.

6. One of the following is a two dimensional vector subspace of $\mathbb{R}^3$. Which one?

   (a) $\{(x,y,z)^t | 2x - y + z = 1\}$,
   (b) $\{(x,y,z)^t | 2x^2 - y + z = 0\}$,
   (c) The column space of $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$,
   (d) The row space of A (see (c)),
   (e) The null space of A (i.e. $\{X | AX = 0\}$) (see (c)).

8. The three points $P(1,0,1)$, $Q(2,4,2)$, $R(-3,2,1)$ determine a plane of equation

(a) $x-1=\frac{y}{4}=\frac{z}{-3}$,

(b) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix} + t \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix}$; $s,t \in \mathbb{R}$,

(c) $x+2y-9z=-8$,

(d) $x-y-z=0$,

(e) $2x+y+2z=4$.

9. Let A be a 2×2 matrix having eigenvalues $\lambda_1=-2$ and $\lambda_3=3$ with corresponding eigenvectors $\mathbf{v}_2=\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\mathbf{v}_3=\begin{pmatrix} -1 \\ 1/2 \end{pmatrix}$. Then A can be only one of the matrices below. Which?

(a) $\begin{pmatrix} -2 & 0 \\ 0 & 3 \end{pmatrix}$, (b) $\begin{pmatrix} -2 & -5 \\ 0 & 3 \end{pmatrix}$, (c) $\begin{pmatrix} -2 & -2 \\ -2 & 3 \end{pmatrix}$,

(d) $\begin{pmatrix} 2 & -2 \\ -2 & -1 \end{pmatrix}$, (e) $\begin{pmatrix} 1 & -1 \\ 2 & 1/2 \end{pmatrix}$.

END OF MULTIPLE CHOICE SECTION

PART II. **Group 1**

(16) 1. Let A be the 5×4 matrix

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ -1 & -2 & 0 & -1 \\ -2 & -4 & -3 & -4 \\ 1 & 2 & 0 & 3 \\ 3 & 6 & 1 & 9 \end{bmatrix}$$

and let I be the 5×5 identity matrix. Using elementary row operations the 5×9 matrix $[I|A]$ can be reduced to a 5×9 matrix $[C|D]$ where

$$[C|D] = \left[\begin{array}{rrrrr|rrrr} 0 & -1 & 0 & 0 & 0 & 1 & 2 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 3 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \\ -1 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 5 \end{array}\right]$$

C is 5×5 and D is 5×4.

(a) Find a basis for the row space of A.

1. (b) Find a basis for the column space of A, consisting of columns of A.

(c) Find a basis for the null space of A.

1. (d) Find the dependence relations on the columns of A.

(e) Determine whether the vector $\vec{v}_1 = (1,1,1,9,24)^t$ is in the column space of A.

(15) 2. Consider the four points $G(4,6,3)$, $A(1,2,6)$, $B(2,1,6)$ and $C(1,3,7)$ in $\mathbb{R}^3$.

(a) Find an equation of the plane through A, B and C.

(b) Find an distance from G to the plane through ABC.

(c) Find an equation of the line through A and B.

(12) 3. The equation

$$7X^2 + 4XY + 4Y^2 = 4$$

represents a conic in the XY coordinate system.

(a) Find an orthogonal matrix P which brings the conic into standard form by rotating the X and Y axes to X_1 and Y_1 axes.

3. (b) What is the equation of the conic in the X_1Y_1 system?

(c) Make a sketch of the conic in the X_1Y_1 and XY system.

(12) 4. Let

$$A = \begin{bmatrix} 2 & -2 & -1 \\ -2 & 2 & 1 \\ -1 & 1 & 5 \end{bmatrix}.$$

(a) Find the eigenvalues of A. (One of the eigenvalues is 3).

4. (b) Find an orthogonal matrix P which diagonalizes the matrix A. (Recall that the columns of P are eigenvectors of A.)

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-221A

VECTORS, MATRICES & GEOMETRY

Examiner: Professor W. Jonsson
Associate Examiner: Professor D. Sussman

Date: Friday, December 20, 1996
Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

1) There are two parts to this examination. Part I consists of 9 multiple choice questions, each worth 5 marks for a total of $9 \times 5 = 45$.
   Part II consists of 4 questions worth a total of 55 marks. **This gives a total of 100 marks in all**.

2) Answers to part I are to be entered onto the **SHORT** machine readable sheets provided with a soft lead pencil. Answers to part II are to be written in the space provided on the question paper. If more space is needed, use an examination booklet.

3) **This exam is in Group** (1). It is important that the group number for the question paper be entered into the column next to the student number on the computer readable sheet.

4) The question paper, the computer readable sheet and any examination booklet(s) must **all** be handed in and each must have recorded on it your name and student number and the name and number of the course (VECTORS, MATRICES & GEOMETRY - 189-221A). Extra examination booklets will be provided if needed.

5) Calculators are not permitted.

Name:______________________________ Student number:____________