1. Find the indicated anti-derivatives

(a) $\int (1-x)\sqrt{2x-x^2}\,dx.$ (b) $\int \frac{e^x}{(e^x+1)\ln(e^x+1)}\,dx,$

(c) $\int x^2 \arctan x\,dx,$ (d) $\int \frac{\ln x}{\sqrt{x}}\,dx.$

2. (a) Use the method of partial fractions to evaluate

$$\int \frac{x^2+x+1}{(x^2+2x+2)^2}\,dx.$$

(b) Evaluate $\int \tan^6 x\,dx.$

3. (a) Use the second fundamental theorem of calculus to evaluate

$$\frac{d}{dx}\left(\int_{\sqrt{x}}^{x} \frac{e^t}{t}dt\right).$$

(b) Find the area enclosed between the curves

$$y = x^2 - x, \text{ and } y = e^x - 1 \text{ for } 0 \le x \le 1.$$

(c) Find the volume obtained by rotating the region bound by $y = \tan x$, $x = 0$ and $y = 1$ around the x-axis.

4. (a) Find the volume obtained by rotating the region bound by $y = x - x^2$ and $y = 0$, about the line $x = 2$.

(b) Decide if the following integrals converge or diverge. Find the value in case of convergence.

(i) $\int_0^{\pi/4} \frac{\cos x}{(\sin x)^{5/4}}dx,$ (ii) $\int_0^{\infty} x^2 e^{-x}\,dx.$

5. (a) Decide if the following series of numbers converge or diverge.

(i) $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{3^{(\ln k/k^2)}},$ (ii) $\sum_{1}^{\infty}\left(\frac{\ln t}{t}\right)^2,$

(iii) $\sum_{1}^{\infty}\frac{k^k}{(k!)^2},$ (iv) $\sum_{1}^{\infty}\frac{e^{-1/n}}{\sqrt{n}}.$

(b) Show that the series $\sum_{1}^{\infty}(-1)^{n+1}\frac{n!}{(2n)!}$ converges. Find an approximate value of the sum of the series with error not exceeding $\frac{1}{1000}$.

6. (a) Given that $\sin u = \sum_{n=0}^{\infty}(-1)^n \frac{u^{2n+1}}{(2n+1)!}$ for all u, express $\sin(tx^2)$ as a power series involving x and t. Use that series to evaluate $\frac{df}{dt}$ where $f(t) = \int_0^1 \sin(tx^2)dx$.

(b) Find the radius and the interval of convergence of the following power series. Check convergence at end points.

(i) $\sum_{1}^{\infty}\frac{n^{10}}{10^n}x^n$ and (ii) $\sum_{1}^{\infty}\frac{\ln k}{3^k}(x-1)^k$.

7. (a) Sketch the curve with polar equation $r = 1 - \sqrt{2}\sin\theta$. Find the area enclosed by the smaller loop of the above curve.

(b) Find the length of the spinal $r = e^{\theta},\ 0 \le \theta \le \pi$.

(c) Find the area of the surface obtained by rotating the curve $x = \frac{1}{2\sqrt{2}}(y^2 - \ln y)$, $1 \le y \le 2$ around the x-axis.

8. (a) Evaluate $\int\int_D e^{x/y}dA$ where D is the region enclosed by the three lines $x = 0,\ y = 1$ and $y = x$.

(b) Interchange the orders of integration and evaluate

$$\int_0^1 \int_{y^2}^1 y\sin(x^2)dx\,dy.$$

(c) Use integration in polar coordinates to evaluate

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2}dx\,dy.$$

McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-151B

CALCULUS B

Examiner: Professor K.N. GowriSankaran — Date: Tuesday, April 18, 2000
Associate Examiner: Professor W.G. Brown — Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

Calculators are not permitted.
All questions count equally.
(Level of difficulty may vary.

This exam comprises the cover and two pages of questions.