

1. (20 marks) Evaluate each of the following limits or show that it does not exist

(a) $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4}$

(b) $\lim_{(x,y) \rightarrow (0,0)} \sin \frac{xy}{x^2 + y^2}$

(c) $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xyz}{x^2 + 2y^2 + z^2}$

(d) $\lim_{x \rightarrow 0^+} x \ln x$

2. Analyse each of the following functions, identifying intervals of increase, decrease, convexity and concavity. Find all critical points and determine if they are local maximum points or local minimum points. Find all points of inflection.

(i) (10 marks) $f(x) = e^{-\frac{1}{2}x^2}$

(ii) (10 marks) $f(x) = (x + 1)^{\frac{1}{3}}(x - 3)$

3. (20 marks) Find the maximum and minimum values of the function

$$f(x, y) = x^2 + 2xy - 2y^2 + x + 3y$$

on the square given by $0 \leq x \leq 1$, $0 \leq y \leq 1$.

4.

- (i) (3 marks) State the Intermediate Value Theorem.
(ii) (3 marks) State the Mean Value Theorem.
(iii) (7 marks) Show that the equation $e^x = 5 - x$ has exactly one real solution.
(iv) (7 marks) Without making any calculations, explain how you would use Newton's Method to find an approximation to this solution.

5. (20 marks) Use Lagrange multipliers to find the point on the surface

$$\frac{x^3}{2} + \frac{y^3}{3} + \frac{z^3}{4} = 1$$

which is closest to the origin.

6. (i) (10 marks) Find the rate of greatest increase of the function

$$f(x, y, z) = \ln(1 + x^2 - y^2 + z^2)$$

at the point $(1, 1, -1)$ as well as the direction of greatest increase.

- (ii) (10 marks) Find the equations of the tangent plane and normal line to the surface $x^3 + y^4 + z^5 = 3$ at the point $(1, 1, 1)$.

7. In this question r , ω and v are constants. A particle moves in space according to the law

$$x(t) = r \cos \omega t, \quad y(t) = r \sin \omega t, \quad z(t) = vt$$

where t represents the time.

- (i) (6 marks) Find the velocity vector of the particle at time t .
(ii) (6 marks) Find the acceleration vector of the particle at time t .
(iii) (8 marks) Let

$$\varphi(x, y, z) = \frac{1}{\sqrt{(x+r)^2 + y^2 + z^2}}.$$

Use the Chain Rule to calculate $\frac{d}{dt}\varphi(x(t), y(t), z(t))$.

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FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-150A

Ordinary Differential Equations

Examiner: Professor S. W. Drury
Associate Examiner: Professor W. G. Brown

Date: Friday, 11 December 1998
Time: 9: 00 am. – 12: 00 noon

INSTRUCTIONS

All seven questions should be attempted for full credit.

This is a closed book examination.

Write your answers in the booklets provided.

All questions are of equal weight, each is worth 20 marks.

No calculators are allowed.

This exam comprises the cover and 2 pages of questions.