1. Given a matrix $A = \begin{pmatrix} 3 & 1 & -1 \\ 1 & -2 & 1 \\ 4 & 3 & 2 \end{pmatrix}$.

   (a) Evaluate A^{-1}, (b) Evaluate $2A + A^{-1}$, (c) Evaluate A^{-2}.

2. An economy is based on two sectors, steel and agriculture. Production of a dollar's worth of steel requires an input of $0.20 of steel and $0.50 of agriculture. Production of a dollar's worth of agriculture requires an input of $0.30 of steel and $0.60 of agriculture.

   (a) Find the output of each sector that is required to satisfy a final demand of $10 million of steel and $20 million of agriculture.

   (b) If the total output of steel is $30 million and the total output of agriculture is $50 million, find the final demand of each sector.

3. Solve by Gauss-Jordan elimination

$$\begin{array}{rcrcrcrcrr} x_1 & - & x_2 & + & 2x_3 & & & & -2x_5 = & 3 \\ -2x_1 & + & 2x_2 & - & 4x_3 & - & x_4 + & & x_5 = & -5 \\ 3x_1 & - & 3x_2 & + & 7x_3 & + & x_4 - & & 4x_5 = & 6 \end{array}$$

4. Find the maximum of $z = x + 2y$ subject to

$$\begin{aligned} -2x + y &\geq 6 \\ x + y &\leq 15 \\ x, y &\geq 0 \end{aligned}$$

   by first graphing the solution region and then finding the corner points.

5. An insurance company found that on average over a 10-year period, 25% of drivers in a particular community who were involved in an accident one year were also involved in an accident the following year. They also found that only 10% of drivers who were not involved in an accident one year were involved in an accident the following year.

   (a) Find the transition matrix P.

   (b) If 5% of drivers in the community are involved in an accident this year, what is the probability that a driver chosen at random from the community will be involved in an accident next year? Year after next?

   (c) In the long run how much of the population will not be involved in an accident?

6. Sketch the graph of the function

$$f(x) = x^4 - 4x^3 + 1.$$

Give the critical points, points of inflection and the values of x for which the function is concave up and concave down. Give also local maximum or minimum points.

7. Find $\dfrac{dy}{dx}$ in the following examples:

(a) $y = (x+1)^{1/3}(2x+1)^{2/3}$; (b) $y = \dfrac{x}{\sqrt{x^2+1}}$;

(c) $y = \ln\left(\dfrac{x^4}{x-1}\right)$; (d) $y = x^2e^{-x^3+5}$.

8. (a) Find the equation of the tangent line for $9x^2 + 4y^2 = 72$ at $(2,3)$.

   (b) A particle is moving along the hyperbola $xy = 4$. As it passes through the point $(1,4)$ its y-coordinate is increasing at a rate of 3 units per second. How fast is the x-coordinate changing at this instant?

9. A marketing research department found that the relationship between price p (\$/unit) and demand x (units/week) was given by $p = 12 - 2\ln x$, $0 < x < 90$. The weekly revenue can be approximated by $R(x) = xp$. The cost function is given by $C(x) = 3x$. Find the local maximum of the profit function. Justify your answer.

10. A farmer wants to construct a rectangular pen next to a barn 80 metres long, using all of barn as part of one side of the pen. Find the dimension of the pen with the largest area that the farmer can build if 250 metres of fencing material is available.

McGILL UNIVERSITY

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-130A

MATHEMATICS FOR MANAGEMENT I

Examiner: Professor N.G.F. Sancho
Associate Examiner: Professor C. Roth

Date: Tuesday, December 15, 1998
Time: 9:00 A.M. - 12:00 Noon.

INSTRUCTIONS

Faculty approved calculators are allowed.

This exam comprises the cover and 2 pages of questions.