

MARKS

- (20) 1. (a) Evaluate:  $\lim_{x \rightarrow 2} \frac{\sqrt{x+2} - \sqrt{2x}}{x^2 - 2x}$ .
- (b) Evaluate:  $\lim_{x \rightarrow 0} \frac{\sin 3x}{4x + 3x^2}$ .
- (c) Evaluate:  $\lim_{x \rightarrow -2} \left( \frac{3}{x^2 + x - 2} - \frac{1}{x^2 + 3x + 2} \right)$ .
- (d) Solve the inequality:  $\frac{x^2 - 3x + 4}{x^2 - 4x + 3} < 1$ .

- (20) 2. (a) Find the value of  $k$  so that the function

$$f(x) = \begin{cases} x - 2, & x \leq 5 \\ kx - 3, & x > 5 \end{cases}$$

is continuous at  $x = 5$ .

- (b) Solve the following equation for  $x$ :

$$x^2 - 5|x| - 24 = 0.$$

- (c) Let  $f(x)$  and  $g(x)$  be inverse functions with  $f(3) = 2$  and  $f'(3) = 5$ . Which one of the following is true?

(i)  $g'(3) = 5$ , (ii)  $g'(3) = \frac{1}{5}$ , (iii)  $g'(2) = 5$ ,  $g'(2) = \frac{1}{5}$ , (v)  $g'(3) = -5$ .

- (d) Use implicit differentiation to find  $\frac{dy}{dx}$  for  $x^3 + 2x^2y - 3y^3 - 4 = 0$ .

- (20) 3. Find  $\frac{dy}{dx}$  for the following: Do not simplify your answers.

(a)  $y = \frac{3}{\sqrt[3]{x}} + \frac{2}{x^2} + \frac{1}{4x^3} + 4\pi^2$ .

(b)  $y = e^{2x} \cos(x^2 + 4)$ .

(c)  $y = \frac{\sin^{-1} 4x}{4x}$ .

(d)  $y = \ln(\sqrt{1+x^2} + 4)$ .

- (6) 4. (a) Use differentials to find the approximate value of  $\sqrt{103}$ .
- (6) (b) Find the equation of the tangent line to the curve  $y = 5x^2 + 3x - 2$  at the point  $(-1, 0)$ .

**Do only ONE** of the following two problems.

- (8) (c) A ladder 10ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1ft/sec, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6ft from the wall.

**OR**

- (8) (c) A closed box with a square base is to have a volume of 250 cubic feet. The material for the top and bottom of the box costs \$2 per square foot, and the material for the sides costs \$1 per square foot. Find the dimensions of the box so that the cost will be a minimum.

- (16) 5. (a) Sketch the graph of the function

$$y = \frac{1}{4}(3x^5 - 20x^3)$$

clearly identifying relative maxima, relative minima, horizontal points of inflection and other points of inflection.

- (4) (b) Find the horizontal and vertical asymptote of the graph of the function  $f(x) = \frac{2x + 1}{3x - 2}$ . Do not sketch the graph of the function.

FACULTY OF SCIENCE

FINAL EXAMINATION

MATHEMATICS 189-120A

CALCULUS I

Examiner: Professor S. Melamed  
Associate Examiner: Professor R. Vermes

Date: Thursday, December 12, 1996  
Time: 9:00 A.M. - 12:00 Noon

INSTRUCTIONS

1. Do all 5 problems in order. Leave blank pages (if necessary) to complete a problem.
2. Write your section number on the top right-hand corner of your examination booklet.
3. Non-programmable calculators are permitted.

This exam comprises the cover and 2 pages of questions.